Victorian Essential Learning Standards

Discipline-based Learning Strand

MATHEMATICS

REVISED EDITION JANUARY 2008

Victoria
The Place To Be

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Contents

Revised Edition January 2008

This edition incorporates minor amendments to the domain introductions and learning focus statements to indicate their relationship with the National Statements of Learning.

Discipline-based Learning

The domains within the Discipline-based Learning strand form a body of knowledge with associated ways of seeing the world and distinct methods of exploring, imagining and constructing that world.

Broadly in line with academic literature and consistent with practice in many schools, the Victorian Essential Learning Standards identify the Arts, the Humanities, English and Languages Other Than English, Mathematics and Science as the disciplines for the curriculum over the stages of learning from Prep to Year 10.

Within the Discipline-based Learning strand the learning domains are:

The Arts
English
The Humanities – Economics
The Humanities – Geography
The Humanities – History
Languages Other Than English (LOTE)
Mathematics
Science

Students who develop a deep understanding of the concepts contained in the discipline-based domains are able to apply their knowledge in many different ways. The degree to which they are able to transfer their knowledge depends largely on the degree to which students have achieved mastery over Physical, Personal and Social and Interdisciplinary learning.

Research suggests that students develop deeper understanding of discipline-based concepts when they are encouraged to reflect on their learning, take personal responsibility for it and relate it to their own world. These approaches are explicitly defined in the Physical, Personal and Social Learning domains such as physical education and personal learning.

Students are better able to develop, demonstrate and use discipline-based knowledge and skills when they are able to employ interdisciplinary knowledge, skills and behaviours described in the domains of Communication; Design, Creativity and Technology; Information and Communications Technology; and Thinking Processes.

Mathematics

Introduction

Mathematics is a human endeavour that has developed by practice and theory from the dawn of civilisation to the present day. Many societies and cultures have contributed to the growth of mathematics, often in times of scientific, technological, artistic and philosophical change and development. Complementary to this broad perspective of mathematics are the various mathematical practices that take place day to day in communities around the world.

While the usefulness of mathematics for modelling and problem solving is well known, mathematics also has a fundamental role in enabling cultural, social and technological advances, and empowering individuals as critical citizens in contemporary society and for the future. Number, space and measurement, and chance and data are common aspects of most people's mathematical experience in everyday personal, study and work situations. Equally important are the essential roles that mathematical structure and working mathematically play in people's understanding of the natural and human worlds.

Mathematics can be described in terms of its objects, what they are and how they came to be; its established body of knowledge and why this is held to be true; its effective application in science, technology and other domains; and the practice and activities of mathematicians past and present. Aims for essential learning in school mathematics are for students to:

- demonstrate useful mathematical and numeracy skills for successful general employment and functioning in society
- solve practical problems with mathematics, especially industry and work-based problems
- develop specialist knowledge in mathematics that provides for further study in the discipline
- see mathematical connections and be able to apply mathematical concepts, skills and processes in posing and solving mathematical problems
- be confident in one's personal knowledge of mathematics, to feel able both to apply it, and to acquire new knowledge and skills when needed
- be empowered through knowledge of mathematics as a numerate citizen, able to apply this knowledge critically in societal and political contexts
- develop understanding of the role of mathematics in life, society and work; the role of mathematics in history; and mathematics as a discipline – its big ideas, history, aesthetics and philosophy.

Mathematical knowledge includes knowledge of concepts, objects, definitions and structures. A small collection of mathematical ideas, objects, structures, and relationships between these, is taken as undefined and given in a context. New mathematical objects, structures and relationships are developed from these moving from simple to more complex and sophisticated ideas and practices. The motivation for accepting certain things as given building blocks for mathematical knowledge may be initially related to intuitive understanding of particular ideas and objects experienced with respect to the natural or human worlds. These and their subsequent developments are not empirical knowledge, but abstract mathematical entities.

Whether mathematical knowledge is viewed as being essentially mind dependent or mind independent, discovered or constructed, its abstract nature gives rise to the applicability of mathematics in a wide range of contexts, as mathematical objects, structures and relationships do not depend on a particular context for their existence, but are interpreted to model key features of these contexts. This abstraction poses a challenge to the teacher and student alike, and both will need to draw on knowledge of the world and link this to mathematical knowledge and its application in various situations.

Mathematical reasoning and thinking underpins all aspects of school mathematics, including problem posing, problem solving, investigation and modelling. It encompasses the development of algorithms for computation, formulation of problems, making and testing conjectures, and the development of abstractions for further investigation.

Computation and proof are essential and complementary aspects of mathematics that enable students to develop thinking skills directed toward explaining, understanding and using mathematical concepts, structures and objects. They provide a framework for the development of mathematical skills and techniques exemplified in the use of algorithms for computation and for the development of general case arguments.

Structure of the domain

The Mathematics domain is organised into six sections, one for each level of achievement from Level 1 to Level 6. Each level includes a learning focus statement and a set of standards organised by dimension. A glossary is included which provides definitions of or additional information about underlined terms (see page 38).

Definitions of underlined terms are provided in the Glossary (page 38)

Learning focus

Learning focus statements are written for each level. These outline the learning that students need to focus on if they are to progress in the domain and achieve the standards at the levels where they apply. They suggest appropriate learning experiences from which teachers can draw to develop relevant teaching and learning activities.

Standards

Standards define what students should know and be able to do at different levels and are written for each dimension. In Mathematics, standards for assessing and reporting on student achievement apply from Level 1. Standards for *Structure* are introduced from Level 3.

Dimensions

Standards in the Mathematics domain are organised in five dimensions:

- Number
- Space
- Measurement, chance and data
- Working mathematically
- Structure.

Number

The *Number* dimension focuses on developing students' understanding of counting, magnitude and order. The natural (counting) numbers with zero extend to positive and negative signed whole numbers (integers) and through part-whole relations and proportions of whole numbers to the rational numbers (fractions and finite decimals or infinite recurring decimals).

Proportions of lengths involving sides and/or diagonals of right-angled triangles and rectangles and arcs of a circle lead to the introduction of certain irrational real numbers such as the square root of 2, the golden ratio *phi* and fractions or multiples of *pi* .

Principal operations for computation with number include various algorithms for addition (aggregation), subtraction (disaggregation) and the related operations of multiplication, division and exponentiation carried out mentally, by hand using written algorithms, and using calculators, spreadsheets or other numeric processors for calculation.

Space

The *Space* dimension focuses on developing students' understanding of shape and location. These are connected through forms of representation

Definitions of underlined terms are provided in the Glossary (page 38)

of two- and three-dimensional objects and the ways in which the shapes of these objects and their ideal representations can be moved or combined through transformations. Students learn about key spatial concepts including continuity, edge, surface, region, boundary, connectedness, symmetry, invariance, congruence and similarity.

Principal operations for computation with space include identification and representation, construction and transformation by hand using drawing instruments, and also by using dynamic geometry technology.

Measurement, chance and data

The *Measurement, chance and data* dimension focuses on developing students' understanding of unit, measure and error, chance and likelihood and inference. Measure is based on the notion of unit (*informal*, *formal* and *standard*) and relates number and natural language to measuring characteristics or attributes of objects and/or events. Various technologies are used to measure, and all measurement involves error.

Students learn important common measures relating to money, length, mass, time and temperature, and probability – the measure of the chance or likelihood of an event. Other measures include area, volume and capacity, weight, angle, and derived rates such as density, concentration and speed.

Principal operations for computation with measurement include the use of formulas for evaluating measures, the use of technology such as dataloggers for direct and indirect measurement and related technologies for the subsequent analysis of data, and estimation of measures using comparison with prior knowledge and experience, and spatial and numerical manipulations.

Structure

The *Structure* dimension focuses on developing students' understanding of set, logic, function and algebra. It is fundamental to the concise and precise nature of mathematics and the generality of its results. Key elements of mathematical structure found in each of the dimensions of Mathematics are membership, operation, closure, identity, inverse, and the commutative, associative and distributive properties as well as other notions such as recursion and periodic behaviour.

While each of these can be considered in its own right, it is in their natural combination as applied to elements of number, space, function, algebra and logic with their characteristic operations that they give rise to the mathematical systems and structures that are embodied in each of these dimensions.

Principal operations for computation with structure include mental, by hand and technology-assisted calculation and symbolic manipulation by calculators, spreadsheets or computer algebra systems, with sets, logic, functions and algebra.

Definitions of underlined terms are provided in the Glossary (page 38)

Working mathematically

Working mathematically focuses on developing students' sense of mathematical inquiry: problem posing and problem solving, modelling and investigation. It involves students in the application of principled reasoning in mathematics, in natural and symbolic language, through the mathematical processes of conjecture, formulation, solution and communication; and also engages them in the aesthetic aspects of mathematics.

In this dimension the nature, purpose and scope of individual work is connected to that of the broader mathematical community, and the historical heritage of mathematics through the discourse of working mathematically. Mental, by hand and technology-assisted methods provide complementary approaches to working mathematically.

Relationships between the dimensions

Number is related to the other dimensions through the aspects of counting, magnitude and order. It has logical and natural connections with *Measurement, chance and data*, and *Space*. Number systems provide the basis for the development of algebraic relationships in *Structure* and the contexts and explorations used in *Working mathematically*.

Space is related to the *Number* and *Measurement, chance and data* dimensions through the aspects of shape and location. The properties of patterns, transformations, and symmetry provide links to *Structure* and *Working mathematically*.

Measurement, chance and data is related to the *Number* and *Space* dimensions through the aspects of units, error, approximation, likelihood, angle, and the properties of two- and three-dimensional shapes. The application of measurement formulas and functions provide a link to *Structure*. A varied collection of practical contexts for generating and testing conjectures provides links to *Working mathematically*.

Structure is related to the *Number*, *Space* and *Measurement, chance and data* dimensions through the use of algorithms, patterns and functions. It is linked to *Working mathematically* through the key elements of mathematical language, concepts and relationships used in modelling and investigations.

Working mathematically is related to the *Number*, *Space* and *Measurement, chance and data* dimensions through the exploration of algorithms, patterns and functions, shapes and dimensions. It provides the processes for the development of inferences and deductions and for the exploration and proof of conjectures related to the *Structure* dimension.

Definitions of underlined terms are provided in the Glossary (page 38)

National Statements of Learning

The Victorian Essential Learning Standards (VELS) incorporate the opportunities to learn covered in the national Statements of Learning (www.curriculum.edu.au/mceetya/the_statements_of_learning,11893.html). The Statements of Learning describe essential skills, knowledge, understandings and capacities that all young Australians should have the opportunity to learn by the end of Years 3, 5, 7 and 9 in English, Mathematics, Science, Civics and Citizenship and Information and Communication Technologies (ICT).

The Statements of Learning were developed as a means of achieving greater national consistency in curriculum outcomes across the eight Australian states and territories. It was proposed that they be used by state and territory departments or curriculum authorities (their primary audience) to guide the future development of relevant curriculum documents. They were agreed to by all states and territories in August 2006.

During 2007, the VCAA prepared a detailed map to show how the Statements of Learning are addressed and incorporated in the VELS. In the majority of cases, the VELS learning focus statements incorporate the Statements of Learning. Some Statements of Learning are covered in more than one domain. In some cases, VELS learning focus statements have been elaborated to address elements of the Statements of Learning not previously specified. These elaborations are noted at the end of each learning focus statement.

National Numeracy Benchmarks

National Numeracy Benchmarks are used for reporting achievement in three aspects of numeracy – 'Number sense', 'Spatial sense' and 'Measurement and data sense' – at Years 3, 5 and 7. The benchmarks define nationally agreed minimum acceptable standards for numeracy at these years.

Full details of the National Numeracy Benchmarks are available in *Numeracy Benchmarks Years 3, 5 and 7*, Curriculum Corporation, 2000 at <http://www.curriculum.edu.au/projects/numbench.php>.

The benchmarks describe minimum standards. For this reason, the Year 3 benchmarks relate to Level 2 Mathematics standards, the Year 5 benchmarks relate to Level 3 Mathematics standards and the Year 7 benchmarks relate to Level 4 Mathematics standards. Links to the numeracy benchmarks are located in the Mathematics standards.

Level 1

Learning focus

As students work towards the achievement of Level 1 standards in Mathematics, they manipulate and play with objects to develop links between their immediate environment, everyday language and mathematical activity.

In *Number*, students manipulate and group physical objects and drawings to develop basic understanding of the concepts of number and numerals. They group objects into sets (collections) and form simple correspondences (relations) between two sets; for example, in sharing pencils among students. They learn to count the number of objects up to 20 and relate the number counted to the use of a numeral. They describe and place objects in order such as first, second and third. They model addition by putting groups of objects together and counting the combined set and they model subtraction by moving apart groups of objects.

In *Space*, students manipulate and investigate the properties of basic two- and three-dimensional shapes. They use everyday objects and drawings to identify and describe points, lines, edges and surfaces. They recognise inside and outside. They participate in activities in which they create and follow simple verbal instructions to locate items in the classroom and immediate environment.

In *Measurement, chance and data*, students learn to compare common objects using terms such as *longer*, *heavier*, *fuller* and *hotter*. They begin to make estimates and simple measurements using informal units such as a number of paper clips in a length. In playing games of chance, students begin to recognise the unpredictability and uncertainty of events such as the roll of a die. They investigate situations requiring data collection and presentation in simple displays such as a pictogram of family pets.

When *Working mathematically*, students undertake activities and play to develop skills in making correspondences (for example, games such as Memory and activities such as matching students with their birth months). They create and explore number patterns using counters or other objects. They take risks by making and exploring conjectures relating to numbers, patterns, shapes and measurements (for example, 'the bigger the object the heavier it is' or 'the next shape in a sequence will be …'). Students work with calculators to check the results of simple addition and subtraction. They draw and copy simple shapes and patterns by hand and also by using a computer drawing package.

Definitions of underlined terms are provided in the Glossary (page 38)

Standards

Number

At Level 1, students form small sets of objects from simple descriptions and make simple correspondences between those sets. They count the size of small sets using the numbers 0 to 20. They use one-to-one correspondence to identify when two sets are equal in size and when one set is larger than another. They form collections of sets of equal size. They use ordinal numbers to describe the position of elements in a set from first to tenth. They use materials to model addition and subtraction by the aggregation (grouping together) and disaggregation (moving apart) of objects. They add and subtract by counting forward and backward using the numbers from 0 to 20.

Space

At Level 1, students recognise, copy and draw points, lines and simple free-hand curves. They identify basic two-dimensional shapes such as triangles, circles and squares and three-dimensional solids and objects such as boxes and balls. They recognise the interior and exterior of shapes and objects. They sort geometric objects according to simple descriptions. They place and orientate shapes according to simple descriptions such as *next to*, *beside*, *in front of*, *behind*, *over* and *under*.

They develop and follow simple instructions to move and place shapes and objects in familiar situations in relation to what they can see, and to move themselves from one place to another.

Measurement, chance and data

At Level 1, students compare length, area, capacity and mass of familiar objects using descriptive terms such as *longer*, *taller*, *larger*, *holds more* and *heavier*. They make measurements using informal units such as paces for length, handprints for area, glasses for capacity, and bricks for weight.

They recognise the continuity of time and the natural cycles such as day/night and the seasons. They correctly sequence days of the week. They use informal units such as heartbeats and hand claps at regular intervals to measure and describe the passage of time.

They recognise and respond to unpredictability and variability in events, such as getting or not getting a certain number on the roll of a die in a game or the outcome of a coin toss. They collect and display data related to their own activities using simple pictographs.

Working mathematically

At Level 1, students use diagrams and materials to investigate mathematical and real life situations. They explore patterns in number and space by manipulating objects according to simple rules (for example, turning letters to make patterns like *bqbqbq*, or flipping to make *bdbdbdbd*).

They test simple conjectures such as 'nine is four more than five'. They make rough estimates and check their work with respect to computations and constructions in *Number*, *Space*, and *Measurement, chance and data*. They devise and follow ways of recording computations using the digit keys and +, – and = keys on a four function calculator.

They use drawing tools such as simple shape templates and geometry software to draw points, lines, shapes and simple patterns. They copy a picture of a simple composite shape such as a child's sketch of a house.

In Mathematics, standards for the *Structure* dimension are introduced at Level 3.

Level 2

Learning focus

As students work towards the achievement of Level 2 standards in Mathematics, they begin to use mathematical symbols and language to describe their mathematical explorations of daily life.

In *Number*, students learn to use base 10 models (units, longs, flats and cubes) and arrays to identify, order and model the counting numbers up to 1000. They create number patterns mentally, by hand and with the use of the constant addition facility of calculators. They use models and arrays to support the development of skip counting up to 100. They recognise patterns created by skip counting (for example, when counting by fours, the pattern of the ones digits is 4, 8, 2, 6, 0, 4, 8). Students perform simple addition (count on) and subtraction (count back) using numbers up to 100. They use equal groups of objects and rectangular arrays to model multiplication and equal sharing for division. Students divide geometric objects including lines, arrays and regular shapes into equal parts to develop the concept of a simple fraction as part of a whole. They learn to order money amounts in dollars and cents, form different totals using dollars and cents, and carry out simple calculations such as change from small amounts.

In *Space*, students participate in activities which focus on identification of key features of shapes and solids. They learn to name familiar two- and three-dimensional shapes. They draw simple two-dimensional shapes, and visualise and describe the effect of transformations (for example, slides, flips and turns). They use mirrors and folding to investigate symmetry of shapes. Students learn to construct and follow directions, informal maps, diagrams and routes to locations in the local environment.

In *Measurement, chance and data*, students learn to use both non-uniform (for example, hand-spans) and uniform (for example, pencil length) informal measurement units. They recognise time units (second, minute, hour, day, week, and month) and investigate basic time patterns and cycles. They learn to tell the time using analogue and digital clocks.

Students pose and respond to questions leading to data collection. They use pictographs and bar graphs to organise and present data. They play games of chance to recognise and quantitatively describe the variability of outcomes. They use terms such as *unlikely* and *almost certain*, *more likely* and *less likely* to describe everyday chance events.

When *Working mathematically*, students learn to use a combination of everyday language and mathematical statements and symbols to describe their manipulation and play with sets of numbers, shapes, objects and patterns. They model and describe daily activities and familiar events using

Definitions of underlined terms are provided in the Glossary (page 38)

physical materials, diagrams and maps (for example, use a 1–1 graph to show attendance at class).

Students test the truth of conjectures by attempting to find examples or counter-examples, and exploring special cases.

They develop and consolidate their understanding of the commutative and associative properties for addition and multiplication. They learn to use a calculator to check estimations, computations and solutions to simple number sentences and equations.

Standards

Number

At Level 2, students model the place value of the natural numbers from 0 to 1000. They order numbers and count to 1000 by 1s, 10s and 100s. Students skip count by 2s, 4s and 5s from 0 to 100 starting from any natural number. They form patterns and sets of numbers based on simple criteria such as odd and even numbers. They order money amounts in dollars and cents and carry out simple money calculations. They describe simple fractions such as one half, one third and one quarter in terms of equal sized parts of a whole object, such as a quarter of a pizza, and subsets such as half of a set of 20 coloured pencils. They add and subtract one- and two-digit numbers by counting on and counting back. They mentally compute simple addition and subtraction calculations involving one- or two-digit natural numbers, using number facts such as complement to 10, doubles and near doubles. They describe and calculate simple multiplication as repeated addition, such as $3 \times 5 = 5 + 5 + 5$; and division as sharing, such as 8 shared between 4. They use commutative and associative properties of addition and multiplication in mental computation (for example, $3 + 4 = 4 + 3$ and $3 + 4 + 5$ can be done as $7 + 5$ or $3 + 9$).

Space

At Level 2, students recognise lines, surfaces and planes, corners and boundaries; familiar two-dimensional shapes including rectangles, rhombuses and hexagons, and three-dimensional shapes and objects including pyramids, cones, and cylinders. They arrange a collection of geometric shapes, such as a set of attribute blocks, into subsets according to simple criteria, and recognise when one set of shapes is a subset of another set of shapes. They recognise and describe symmetry, asymmetry, and congruence in these shapes and objects. They accurately draw simple two-dimensional shapes by hand and construct, copy and combine these shapes using drawing tools and geometry software. They apply simple transformations to shapes (*flips*, turns, slides and enlargements) and depict both the original and transformed shape together. They specify location as a relative position, including left and right, and interpret simple networks, diagrams and maps involving a small number of points, objects or locations.

Definitions of underlined terms are provided in the Glossary (page 38)

Measurement, chance and data

At Level 2, students make, describe and compare measurements of length, area, volume, mass and time using informal units. They recognise the differences between non-uniform measures, such as hand-spans, to measure length, and uniform measures, such as icy-pole sticks. They judge relative capacity of familiar objects and containers by eye and make informal comparisons of weight by hefting. They describe temperature using qualitative terms (for example, cold, warm, hot). Students use formal units such as hour and minute for time, litre for capacity and the standard units of metres, kilograms and seconds.

Students recognise the key elements of the calendar and place in sequence days, weeks and months. They describe common and familiar time patterns and such as the time, duration and day of regular sport training and tell the time at hours and half-hours using an analogue clock, and to hours and minutes using a digital clock.

Students predict the outcome of chance events, such as the rolling of a die, using qualitative terms such as certain, likely, unlikely and impossible. They collect simple categorical and numerical data (count of frequency) and present this data using pictographs and simple bar graphs.

Working mathematically

At Level 2, students make and test simple conjectures by finding examples, counter-examples and special cases and informally decide whether a conjecture is likely to be true. They use place value to enter and read displayed numbers on a calculator. They use a four-function calculator, including use of the constant addition function and × key, to check the accuracy of mental and written estimations and approximations and solutions to simple number sentences and equations.

In Mathematics, standards for the *Structure* dimension are introduced at Level 3.

Year 3 National Numeracy Benchmarks

The benchmarks describe minimum standards. For this reason, the Year 3 benchmarks relate to Level 2 Mathematics standards. Full details of the National Numeracy Benchmarks are available in *Numeracy Benchmarks Years 3, 5 and 7*, Curriculum Corporation, 2000 at <http://www.curriculum.edu.au/projects/numbench.php>.

Level 3

Learning focus

As students work towards the achievement of Level 3 standards in Mathematics, they recognise and explore patterns in numbers and shapes. They increasingly use mathematical terms and symbols to describe computations, measurements and characteristics of objects.

In *Number*, students use structured materials to explore place value and order of numbers to tens of thousands. They skip count to create number patterns. They use materials to develop concepts of decimals to hundredths. They use suitable fraction material to develop concepts of equivalent fractions and to compare fraction sizes. They apply number skills to everyday contexts such as shopping. They extend addition and subtraction computations to three digit numbers. They learn to multiply and divide by single digit numbers.

In *Space*, students sort lines, shapes and solids according to key features. They use nets to create three-dimensional shapes and explore them by counting edges, faces and vertices. They visualise and draw simple solids as they appear from different positions. They investigate simple transformations (reflections, slides and turns) to create tessellations and designs. They explore the concept of angle as turn (for example, using clock hands) and as parts of shapes and objects (for example, at the vertices of polygons). They use grid references (for example, A5 on a street directory) to specify location and compass bearings to describe directions. They use local and larger-scale maps to locate places and describe suitable routes between them.

In *Measurement, chance and data*, students measure the attributes of everyday objects and events using formal (for example, metres and centimetres) and informal units (for example, pencil lengths). Students tell the time using analogue and digital clocks and relate familiar activities to the calendar. Students investigate natural variability in chance events and order them from least likely to most likely. Students conduct experiments and collect data to construct simple frequency graphs. They use simple two-way tables (karnaugh maps) to sort non-numerical data.

In *Structure*, students use structured material (in tens, hundreds and thousands) to develop ideas about multiplication by replication and division by sharing. They recognise the possibility of remainders when dividing. They learn to use number properties to support computations (for example, they use the commutative and associative properties for adding or multiplying three numbers in any order or combination). They investigate the distributive property to develop methods of multiplication and division by single digit whole numbers. They learn to use and describe simple algorithms for computations. They use simple rules to generate number patterns (for

Definitions of underlined terms are provided in the Glossary (page 38)

example, 'the next term in the sequence is two more than the previous term'). They create and complete number sentences using whole numbers, decimals and fractions.

When *Working mathematically*, students use mathematical symbols (for example, brackets, division and inequality, the words and, or and not). Students develop and test ideas (conjectures) across the content of mathematical experience. For example:

- in *Number*, the size and type of numbers resulting from computations
- in *Space*, the effects of transformations of shapes
- in *Measurement, chance and data*, the outcomes of random experiments and inferences from collected samples.

Students learn to recognise practical applications of mathematics in daily life, including shopping, travel and time of day. They identify the mathematical nature of problems for investigation. They choose and use learned facts, procedures and strategies to find solutions. They use a range of tools for mathematical work, including calculators, computer drawing packages and measuring tools.

National Statements of Learning

This learning focus statement, with the following elaboration, incorporates the Year 3 National Statement of Learning for Mathematics.

Elaboration:

They recognise angles … as parts of shapes and objects …

Standards

Number

At Level 3, students use place value (as the idea that 'ten of these is one of those') to determine the size and order of whole numbers to tens of thousands, and decimals to hundredths. They round numbers up and down to the nearest unit, ten, hundred, or thousand. They develop fraction notation and compare simple common fractions such as $\frac{3}{4} > \frac{2}{3}$ using physical models. They skip count forwards and backwards, from various starting points using multiples of 2, 3, 4, 5, 10 and 100.

They estimate the results of computations and recognise whether these are likely to be over-estimates or under-estimates. They compute with numbers up to 30 using all four operations. They provide automatic recall of multiplication facts up to 10 × 10.

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They devise and use written methods for:

- whole number problems of addition and subtraction involving numbers up to 999
- multiplication by single digits (using recall of multiplication tables) and multiples and powers of ten (for example, 5 × 100, 5 × 70)
- division by a single-digit divisor (based on inverse relations in multiplication tables).

They devise and use algorithms for the addition and subtraction of numbers to two decimal places, including situations involving money. They add and subtract simple common fractions with the assistance of physical models.

Space

At Level 3, students recognise and describe the directions of lines as vertical, horizontal or diagonal. They recognise angles are the result of rotation of lines with a common end-point. They recognise and describe polygons. They recognise and name common three-dimensional shapes such as spheres, prisms and pyramids. They identify edges, vertices and faces. They use two-dimensional nets, cross-sections and simple projections to represent simple three-dimensional shapes. They follow instructions to produce simple tessellations (for example, with triangles, rectangles, hexagons) and puzzles such as tangrams. They locate and identify places on maps and diagrams. They give travel directions and describe positions using simple compass directions (for example, N for North) and grid references on a street directory.

Measurement, chance and data

At Level 3, students estimate and measure length, area, volume, capacity, mass and time using appropriate instruments. They recognise and use different units of measurement including informal (for example, paces), formal (for example, centimetres) and standard metric measures (for example, metre) in appropriate contexts. They read linear scales (for example, tape measures) and circular scales (for example, bathroom scales) in measurement contexts. They read digital time displays and analogue clock times at five-minute intervals. They interpret timetables and calendars in relation to familiar events. They compare the likelihood of everyday events (for example, the chances of rain and snow). They describe the fairness of events in qualitative terms. They plan and conduct chance experiments (for example, using colours on a spinner) and display the results of these experiments. They recognise different types of data: non-numerical (categories), separate numbers (discrete), or points on an unbroken number line (continuous).They use a column or bar graph to display the results of an experiment (for example, the frequencies of possible categories).

Structure

At Level 3, students recognise that the sharing of a collection into equal-sized parts (division) frequently leaves a remainder. They investigate sequences of decimal numbers generated using multiplication or division by 10. They understand the meaning of the '=' in mathematical statements and technology displays (for example, to indicate either the result of a computation or equivalence). They use number properties in combination to facilitate computations (for example, $7 + 10 + 13 = 10 + 7 + 13 = 10 + 20$). They multiply using the distributive property of multiplication over addition (for example, $13 \times 5 = (10 + 3) \times 5 = 10 \times 5 + 3 \times 5$). They list all possible outcomes of a simple chance event. They use lists, venn diagrams and grids to show the possible combinations of two attributes. They recognise samples as subsets of the population under consideration (for example, pets owned by class members as a subset of pets owned by all children). They construct number sentences with missing numbers and solve them.

Working mathematically

At Level 3, students apply number skills to everyday contexts such as shopping, with appropriate rounding to the nearest five cents. They recognise the mathematical structure of problems and use appropriate strategies (for example, recognition of sameness, difference and repetition) to find solutions.

Students test the truth of mathematical statements and generalisations. For example, in:

- number (which shapes can be easily used to show fractions)
- computations (whether products will be odd or even, the patterns of remainders from division)
- number patterns (the patterns of ones digits of multiples, terminating or repeating decimals resulting from division)
- shape properties (which shapes have symmetry, which solids can be stacked)
- transformations (the effects of slides, reflections and turns on a shape)
- measurement (the relationship between size and capacity of a container).

Students use calculators to explore number patterns and check the accuracy of estimations. They use a variety of computer software to create diagrams, shapes, tessellations and to organise and present data.

Year 5 National Numeracy Benchmarks

The benchmarks describe minimum standards. For this reason, the Year 5 benchmarks relate to Level 3 Mathematics standards. Numeracy benchmarks are located at Curriculum Corporation <http://www.curriculum.edu.au/projects/numbench.php>.

Level 4

Learning focus

As students work towards the achievement of Level 4 standards in Mathematics, they describe their investigations with correct mathematical terms, symbols and notations. They use mathematical procedures to construct and systematically investigate conjectures or hypotheses.

In *Number*, students extend their understanding of whole numbers, fractions and decimals. They use patterns and arrays to develop understanding of multiples (including lowest common multiple), factors (including highest common factor), prime and composite numbers. They recognise and use simple powers (for example, $2^3 = 8$).

Students investigate and use equivalent forms of common fractions. They order fractions and decimals and locate them on a number line. They investigate temperature and other contexts to develop the concept of negative numbers. They explore ideas of ratio (as a comparison) and percentage (comparing to 100). They use materials to explore decimals, ratios and percentages as equivalent forms of fractions (for example, $\frac{1}{2} = 0.5 = 50\% = 1 : 2$).

Students devise and use mental and written methods (algorithms) to add, subtract, multiply and divide whole numbers. For division they recognise remainders as common fractions or decimals. They devise and use mental and written methods to add and subtract decimals. They use materials and number lines to develop understanding of multiplication and division of decimals (to two decimal places) and simple common fractions. They routinely make estimations and approximations in calculations and make judgments about their accuracy.

In *Space*, students identify and sort shapes by properties such as parallel and perpendicular lines (for example, quadrilaterals). They use the ideas of angle, size and scale to describe the features of shapes and solids. They identify symmetry by reflection or rotation. They create and compare pairs of enlarged shapes using simple scale factors. They describe the features that change (for example, side lengths) and features that remain the same (for example, angles). They represent solids (for example, prisms, pyramids, cylinders and cones) as two-dimensional drawings and nets. They visualise and describe relative location and routes between places shown on a map. They create and interpret simple networks such as a road network to show connectedness between towns.

In *Measurement, chance and data*, students estimate and measure lengths (including perimeter), area (including surface area), volumes, capacity, time (including duration), and temperature in metric units using appropriate instruments and scales. They determine and use the level of accuracy required

Definitions of underlined terms are provided in the Glossary (page 38)

for the purpose of the measurement. They develop simple procedures to determine the perimeter and area of simple shapes (for example, counting squares in a grid to determine area).

Students estimate and describe the chance of random events using words, percentages and fractions or decimals between 0 and 1. They investigate the sample space (possible outcomes) for simple chance events and calculate theoretical probability. They explain how symmetry in chance situations (for example, the roll of a die) creates equally likely outcomes. They create simulations of chance events to estimate probability (for example, randomly selecting a card from a pack without kings to choose a month).

Students plan and conduct questionnaires to collect data for a specific purpose. They recognise different data types such as categorical and numerical, discrete and continuous. They organise and present grouped and ungrouped data using displays such as simple frequency tables and histograms. They calculate and interpret measures of centre (mean, median and mode) and spread (range) for ungrouped data.

In *Structure*, students use venn diagrams and tables (karnaugh maps) to test the validity of statements involving the quantifiers *none*, *some* and *all*. They develop algorithms involving words, diagrams and mathematical symbols (for example, for testing the divisibility of a number).

Students create number sequences by computing the next term from the previous term or terms (recursion). They develop function rules for the terms in sequences based on their position in the sequence.

Students recognise that the ‘identity’ for each operation has no effect: the number 0 for addition and subtraction, and 1 for multiplication and division. They form and solve equations using words and symbols.

When *Working mathematically*, students make and test conjectures and generalisations about numbers, shapes and mathematical structure using concrete materials and diagrams. For example:

- in *Number*, the factors of primes and composites
- in *Space*, the properties of shapes
- in *Measurement, chance and data*, the probability of outcomes in games of chance
- in *Structure*, the patterns of remainders formed by division.

Students identify and investigate real life, practical and historical applications of mathematics. They pose and solve mathematical problems using a range of strategies (for example, make a list, find a pattern, work backwards). They solve new problems based on familiar problem structures.

Definitions of underlined terms are provided in the Glossary (page 38)

Students develop and use estimation procedures to check the results of computations made using technology. They use technology for complex and extended computations. They use appropriate technology to explore puzzles involving numbers (for example, solve a magic square using a spreadsheet) and to generate drawings of shapes, solids, nets and geometric designs.

National Statements of Learning

This learning focus statement, with the following elaboration, incorporates the Year 5 National Statement of Learning for Mathematics.

Standards

Number

At Level 4, students comprehend the size and order of small numbers (to thousandths) and large numbers (to millions). They model integers (positive and negative whole numbers and zero), common fractions and decimals. They place integers, decimals and common fractions on a number line. They create sets of number multiples to find the lowest common multiple of the numbers. They interpret numbers and their factors in terms of the area and dimensions of rectangular arrays (for example, the factors of 12 can be found by making rectangles of dimensions 1 × 12, 2 × 6, and 3 × 4).

Students identify square, prime and composite numbers. They create factor sets (for example, using factor trees) and identify the highest common factor of two or more numbers. They recognise and calculate simple powers of whole numbers (for example, $2^4 = 16$).

Students use decimals, ratios and percentages to find equivalent representations of common fractions (for example, $\frac{3}{4} = \frac{9}{12} = 0.75 = 75\% = 3 : 4 = 6 : 8$). They explain and use mental and written algorithms for the addition, subtraction, multiplication and division of natural numbers (positive whole numbers). They add, subtract, and multiply fractions and decimals (to two decimal places) and apply these operations in practical contexts, including the use of money. They use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Space

At Level 4, students classify and sort shapes and solids (for example, prisms, pyramids, cylinders and cones) using the properties of lines (orientation and size), angles (less than, equal to, or greater than 90°), and surfaces. They create two-dimensional representations of three dimensional shapes and objects found in the surrounding environment. They develop and follow instructions to draw shapes and nets of solids using simple scale. They describe the features of shapes and solids that remain the same (for example, angles) or change (for example, surface area) when a shape is enlarged or reduced. They apply a range of transformations to shapes and create tessellations using tools (for example, computer software).

Students use the ideas of size, scale, and direction to describe relative location and objects in maps. They use compass directions, coordinates, scale and distance, and conventional symbols to describe routes between places shown on maps. Students use network diagrams to show relationships and connectedness such as a family tree and the shortest path between towns on a map.

Measurement, chance and data

At Level 4, students use metric units to estimate and measure length, perimeter, area, surface area, mass, volume, capacity, time and temperature. They measure angles in degrees. They measure as accurately as needed for the purpose of the activity. They convert between metric units of length, capacity and time (for example, L–mL, sec–min).

Students describe and calculate probabilities using words, and fractions and decimals between 0 and 1. They calculate probabilities for chance outcomes (for example, using spinners) and use the symmetry properties of equally likely outcomes. They simulate chance events (for example, the chance that a family has three girls in a row) and understand that experimental estimates of probabilities converge to the theoretical probability in the long run.

Students recognise and give consideration to different data types in forming questionnaires and sampling. They distinguish between categorical and numerical data and classify numerical data as discrete (from counting) or continuous (from measurement). They present data in appropriate displays (for example, a pie chart for eye colour data and a histogram for grouped data of student heights). They calculate and interpret measures of centrality (mean, median, and mode) and data spread (range).

Structure

At Level 4 students form and specify sets of numbers, shapes and objects according to given criteria and conditions (for example, 6, 12, 18, 24 are the even numbers less than 30 that are also multiples of three). They use venn diagrams and karnaugh maps to test the validity of statements using the words *none*, *some* or *all* (for example, test the statement '*all* the multiples of 3, less than 30, are even numbers').

Students construct and use rules for sequences based on the previous term, recursion (for example, the next term is three times the last term plus two), and by formula (for example, a term is three times its position in the sequence plus two).

Students establish equivalence relationships between mathematical expressions using properties such as the distributive property for multiplication over addition (for example, $3 \times 26 = 3 \times (20 + 6)$).

Students identify relationships between variables and describe them with language and words (for example, how hunger varies with time of the day).

Students recognise that addition and subtraction, and multiplication and division are inverse operations. They use words and symbols to form simple equations. They solve equations by trial and error.

Working mathematically

At Level 4, use students recognise and investigate the use of mathematics in real (for example, determination of test results as a percentage) and historical situations (for example, the emergence of negative numbers).

Students develop and test conjectures. They understand that a few successful examples are not sufficient proof and recognise that a single counter-example is sufficient to invalidate a conjecture. For example, in:

- number (all numbers can be shown as a rectangular array)
- computations (multiplication leads to a larger number)
- number patterns (the next number in the sequence 2, 4, 6 … must be 8)
- shape properties (all parallelograms are rectangles)
- chance (a six is harder to roll on die than a one).

Students use the mathematical structure of problems to choose strategies for solutions. They explain their reasoning and procedures and interpret solutions. They create new problems based on familiar problem structures.

Students engage in investigations involving mathematical modelling. They use calculators and computers to investigate and implement algorithms (for example, for finding the lowest common multiple of two numbers), explore number facts and puzzles, generate simulations (for example, the gender of children in a family of four children), and transform shapes and solids.

Year 7 National Numeracy Benchmarks

The benchmarks describe minimum standards. For this reason, the Year 7 benchmarks relate to Level 4 Mathematics standards. Numeracy benchmarks are located at Curriculum Corporation <http://www.curriculum.edu.au/projects/numbench.php>

Level 5

Learning focus

As students work towards the achievement of Level 5 standards in Mathematics, they construct mathematical models to explore and describe the physical world. They recognise the importance of mathematics in a technological society.

In *Number*, students investigate and explore whole numbers and fractions as squares, square roots and other simple powers. They express natural numbers as products of prime number factors.

Students use number lines and materials to compare quantities using ratios, and to form equal ratios using proportion. They use ratios of number pairs to understand constant rate of change. They use number lines, graphs, numerical or algebraic means to solve proportion problems and percentage problems as proportion relative to 100.

Students use patterns with division to develop understanding of infinite decimals, and recognise the existence and applications of non-repeating infinite decimals (for example, π). Students use mental, written or calculator methods for computations, including multiple operations using rounding and estimation to provide suitable answers for practical situations. They use materials and patterns to understand binary numbers and to add and subtract using this notation.

In *Space*, students construct shapes and regular polygons to given specifications. They explore the properties of angles formed by intersecting straight lines. They use ideas of congruency and similarity to create and describe designs and tessellations. They use nets and isometric diagrams for common three-dimensional shapes to construct corresponding geometric objects. They use perspective to draw three-dimensional objects on paper.

Students interpret and use a range of familiar and common maps of locations from small to large scale, using plans and grids. They explore the patterns formed by following procedures involving simple transformations or movements around grids. They use networks to represent relationships in everyday life (for example, a tree diagram for a family tree and a network to show the route used to travel to school).

In *Measurement, chance and data*, students use metric units to estimate and measure length, perimeter, area, surface area, mass, volume, capacity, angle in shapes and solids, time, and temperature. They convert metric units into smaller or larger units as required. They judge the accuracy of their estimates by measurement and calculate error. They use mensuration formulas (for example, for area and perimeter of circles, area and perimeter of triangles and

Definitions of underlined terms are provided in the Glossary (page 38)

parallelograms, and the surface area and volume of prisms and cylinders). They solve problems involving simple rates (per unit time or area).

Students estimate probability from simulations involving generation of random numbers and data of long-run frequencies. They calculate theoretical probabilities involving one- and two-event trials.

Students take samples in order to make inferences and predictions about a population. They learn to present data in appropriate graphical formats. They calculate and interpret summary statistics (mean, median, mode and range).

In *Structure*, students use diagrams to show the relationships between natural, integer, rational and irrational numbers. They give examples of the use of number properties (commutative, associative and distributive) and use counter-examples to show where they do not apply. They test logical equivalence of sentences using the quantifiers *none*, *some* and *all* and set operations of complement, intersection and union, by means of diagrams.

Students use the opposite of any integer for addition, and the inverse of any rational number for multiplication (reciprocal) to rearrange formulas and simple algebraic expressions and to solve equations. They use linear and other simple functions of a single variable, to explore number patterns and provide models for practical situations. They represent functions by tables of values, ordered pairs, graphs and rules applied over a given domain. They solve equations and inequalities with a sequence of inverse operations.

When *Working mathematically*, students determine different but equivalent ways to describe a set, using attributes linked by *and*, *or*, *not*, and by ideas of *implication* and *equivalence*. They generalise from multiple examples and informally justify those generalisations. They use linear and other simple mathematical models to explore practical situations. They make and test predictions from these models (including interpolation and extrapolation). They use technologies such as geometry software, graphics calculators and spreadsheets.

National Statements of Learning

This learning focus statement, with the following elaboration, incorporates the Year 7 National Statement of Learning for Mathematics.

Elaboration:

They construct three-dimensional objects from … isometric diagrams.

Definitions of underlined terms are provided in the Glossary (page 38)

Standards

Number

At Level 5, students identify complete factor sets for natural numbers and express these natural numbers as products of powers of primes (for example, $36\ 000 = 2^5 \times 3^2 \times 5^3$).

They write equivalent fractions for a fraction given in simplest form (for example, $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \ldots$). They know the decimal equivalents for the unit fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{9}$ and find equivalent representations of fractions as decimals, ratios and percentages (for example, a subset: set ratio of 4:9 can be expressed equivalently as $\frac{4}{9} = 0.\overline{4} \approx 44.44\%$). They write the reciprocal of any fraction and calculate the decimal equivalent to a given degree of accuracy.

Students use knowledge of perfect squares when calculating and estimating squares and square roots of numbers (for example, $20^2 = 400$ and $30^2 = 900$ so $\sqrt{700}$ is between 20 and 30). They evaluate natural numbers and simple fractions given in base-exponent form (for example, $5^4 = 625$ and $(\frac{2}{3})^2 = \frac{4}{9}$). They know simple powers of 2, 3, and 5 (for example, $2^6 = 64$, $3^4 = 81$, $5^3 = 125$). They calculate squares and square roots of rational numbers that are perfect squares (for example, $\sqrt{0.81} = 0.9$ and $\frac{\sqrt{9}}{16} = \frac{3}{4}$). They calculate cubes and cube roots of perfect cubes (for example, $\sqrt[3]{64} = 4$). Using technology they find square and cube roots of rational numbers to a specified degree of accuracy (for example, $\sqrt[3]{200} = 5.848$ to three decimal places).

Students express natural numbers base 10 in binary form, (for example, $42_{10} = 101010_2$), and add and multiply natural numbers in binary form (for example, $101_2 + 11_2 = 1000_2$ and $101_2 \times 11_2 = 1111_2$).

Students understand ratio as both set: set comparison (for example, number of boys : number of girls) and subset: set comparison (for example, number of girls : number of students), and find integer proportions of these, including percentages (for example, the ratio number of girls: the number of boys is 2 : 3 = 4 : 6 = 40% : 60%).

Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding (for example, $925 \div 34 \approx 900 \div 30 = 30$).

Students use efficient mental and/or written methods for arithmetic computation involving rational numbers, including division of integers by two-digit divisors. They use approximations to π in related measurement calculations (for example, $\pi \times 5^2 = 25\pi = 78.54$ correct to two decimal places).

They use technology for arithmetic computations involving several operations on rational numbers of any size.

Space

At Level 5, students construct two-dimensional and simple three-dimensional shapes according to specifications of length, angle and adjacency. They use the properties of parallel lines and transversals of these lines to calculate angles that are supplementary, corresponding, allied (co-interior) and alternate. They describe and apply the angle properties of regular and irregular polygons, in particular, triangles and quadrilaterals. They use two-dimensional nets to construct a simple three-dimensional object such as a prism or a platonic solid. They recognise congruence of shapes and solids. They relate similarity to enlargement from a common fixed point. They use single-point perspective to make a two-dimensional representation of a simple three-dimensional object. They make tessellations from simple shapes.

Students use coordinates to identify position in the plane. They use lines, grids, contours, isobars, scales and bearings to specify location and direction on plans and maps. They use network diagrams to specify relationships. They consider the connectedness of a network, such as the ability to travel through a set of roads between towns.

Measurement, chance and data

At Level 5, students measure length, perimeter, area, surface area, mass, volume, capacity, angle, time and temperature using suitable units for these measurements in context. They interpret and use measurement formulas for the area and perimeter of circles, triangles and parallelograms and simple composite shapes. They calculate the surface area and volume of prisms and cylinders.

Students estimate the accuracy of measurements and give suitable lower and upper bounds for measurement values. They calculate absolute percentage error of estimated values.

Students use appropriate technology to generate random numbers in the conduct of simple simulations.

Students identify empirical probability as long-run relative frequency. They calculate theoretical probabilities by dividing the number of possible successful outcomes by the total number of possible outcomes. They use tree diagrams to investigate the probability of outcomes in simple multiple event trials.

Students organise, tabulate and display discrete and continuous data (grouped and ungrouped) using technology for larger data sets. They represent univariate data in appropriate graphical forms including dot plots, stem and leaf plots, column graphs, bar charts and histograms. They calculate summary statistics for measures of centre (mean, median, mode) and spread (range, and mean absolute difference), and make simple inferences based on this data.

Structure

At Level 5, students identify collections of numbers as subsets of natural numbers, integers, rational numbers and real numbers. They use venn diagrams and tree diagrams to show the relationships of intersection, union, inclusion (subset) and complement between the sets. They list the elements of the set of all subsets (power set) of a given finite set and comprehend the partial-order relationship between these subsets with respect to inclusion (for example, given the set $\{a, b, c\}$ the corresponding power set is $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$).

They test the validity of statements formed by the use of the connectives *and*, *or*, *not*, and the quantifiers *none*, *some* and *all*, (for example, '*some* natural numbers can be expressed as the sum of two squares'). They apply these to the specification of sets defined in terms of one or two attributes, and to searches in data-bases.

Students apply the commutative, associative, and distributive properties in mental and written computation (for example, 24×60 can be calculated as $20 \times 60 + 4 \times 60$ or as $12 \times 12 \times 10$). They use exponent laws for multiplication and division of power terms (for example $2^3 \times 2^5 = 2^8$, $2^0 = 1$, $2^3 \div 2^5 = 2^{-2}$, $(5^2)^3 = 5^6$ and $(3 \times 4)^2 = 3^2 \times 4^2$).

Students generalise from perfect square and difference of two square number patterns (for example, $25^2 = (20 + 5)^2 = 400 + 2 \times (100) + 25 = 625$. And $35 \times 25 = (30 + 5)(30 - 5) = 900 - 25 = 875$)

Students recognise and apply simple geometric transformations of the plane such as translation, reflection, rotation and dilation and combinations of the above, including their inverses.

They identify the identity element and inverse of rational numbers for the operations of addition and multiplication (for example, $\frac{1}{2} + \frac{-1}{2} = 0$ and $\frac{2}{3} \times \frac{3}{2} = 1$).

Students use inverses to rearrange simple mensuration formulas, and to find equivalent algebraic expressions (for example, if $P = 2L + 2W$, then $W = \frac{P}{2} - L$. If $A = \pi r^2$ then $r = \frac{\sqrt{A}}{\neq}$ for $r > 0$).

They solve simple equations (for example, $5x + 7 = 23$, $1.4x - 1.6 = 8.3$, and $4x^2 - 3 = 13$) using tables, graphs and inverse operations. They recognise and use inequality symbols. They solve simple inequalities such as $y \leq 2x + 4$ and decide whether inequalities such as $x^2 > 2y$ are satisfied or not for specific values of x and y.

Students identify a function as a one-to-one correspondence or a many-to-one correspondence between two sets. They represent a function by a table of values, a graph, and by a rule. They describe and specify the independent variable of a function and its domain , and the dependent variable and its range. They construct tables of values and graphs for linear functions. They use linear and other functions such as $f(x) = 2x – 4$, $xy = 24$, $y = 2^x$ and $y = x^2 – 3$ to model various situations.

Working mathematically

At Level 5, students formulate conjectures and follow simple mathematical deductions (for example, if the side length of a cube is doubled, then the surface area increases by a factor of four, and the volume increases by a factor of eight).

Students use variables in general mathematical statements. They substitute numbers for variables (for example, in equations, inequalities, identities and formulas).

Students explain geometric propositions (for example, by varying the location of key points and/or lines in a construction).

Students develop simple mathematical models for real situations (for example, using constant rates of change for linear models). They develop generalisations by abstracting the features from situations and expressing these in words and symbols. They predict using interpolation (working with what is already known) and extrapolation (working beyond what is already known). They analyse the reasonableness of points of view, procedures and results, according to given criteria, and identify limitations and/or constraints in context.

Students use technology such as graphic calculators, spreadsheets, dynamic geometry software and computer algebra systems for a range of mathematical purposes including numerical computation, graphing, investigation of patterns and relations for algebraic expressions, and the production of geometric drawings.

Level 6

Learning focus

As students work towards the achievement of Level 6 standards in Mathematics, they extend their use of mathematical models to a wide range of familiar and unfamiliar contexts. They recognise the role of logical argument and proof in establishing mathematical propositions.

In *Number*, students investigate familiar and unfamiliar situations and contexts involving the use of all types of real numbers. They use irrational numbers such as φ , π, and common surds in calculations in both exact and approximate form. They apply mental, written or technology-assisted forms of computation as appropriate, using estimation to validate their answers. They compute using large or small numbers expressed in scientific notation. They evaluate and use factorials in relevant contexts. They apply the concepts of rounding to either a given number of decimal places or significant figures to check the accuracy of computations.

In *Space*, students investigate the possible orientation of lines in space. They investigate the properties of angles formed when lines (including parallel lines) intersect. They learn how space is enclosed in two and three dimensions, and systematically investigate the properties of boundaries and regions on surfaces with shapes such as polygons and circles, prisms and polyhedra (including the platonic solids). They learn to use the concepts of congruency and similarity to compare the size and shape of polygons. They investigate the properties of similar triangles.

Students investigate the relationship between position, length and angle using the pythagorean relationship and trigonometry of right-angled triangles. They explore simple combinations of rotations, translations and reflections as transformations of geometric shapes in the plane. They investigate the paths (loci) formed by points, lines and shapes as they move in space according to various rules, conditions and/or constraints involving transformations. They use symmetry and other properties to create tessellations in two and three dimensions from regular and composite shapes. They investigate the effects of changing the scale of one characteristic of a geometric shape (for example, length or angle) on the size of related characteristics (for example, area and volume).

Students use maps and globes to investigate location and distances between places.

In *Measurement, chance and data*, students measure and estimate perimeter, area, surface area, mass, volume, capacity, angle, and the rates of speed, density and concentration. They use and convert units to suit the purpose of measurements. They make judgments about errors in measurement. They

Definitions of underlined terms are provided in the Glossary (page 38)

use formulas (including trigonometry) to calculate perimeters, areas, angles in shapes, and the surface areas and volumes of solids. They use degrees and radians, as applicable, for units of measurement of angles.

Students apply probability concepts to aspects of chance and risk in everyday life. They represent event spaces that show the nature of events and their probabilities, and use these representations to assist in the computation of the probabilities of compound, independent and dependent events. They apply the concept of mathematical expectation to describe expected gain or loss in games of chance.

Students collect and use uni-variate and bi-variate data samples. They select appropriate representations to display data distributions, centrality, spread, and association between bi-variate data sets.

In *Structure*, students learn to categorise natural, integer, rational and irrational numbers in relation to real numbers. They use the concepts of order, discrete and continuous, and finite and infinite in relation to these sets of numbers.

Students apply algebraic properties (for example, closure, associative, commutative, identity, inverse and distributive) to expressions, formulas and equations.

They relate sets with one, two or three attributes, in four ways:

- diagrams and grids
- the logical connectives *and*, *or*, *not*, *implication* and *equivalence*
- the quantifiers *none*, *some* and *all*
- the set operations *complement*, *intersection*, *union* and *inclusion*.

Students work with functions (for example, linear, quadratic, reciprocal, exponential), simple transformations of these functions, their graphs and related algebraic properties. They solve equations of the form $f(x) = k$, where k is a real constant. They solve simultaneous linear equations using algebraic, numerical and graphical approaches.

When *Working mathematically*, students develop generalisations by abstracting the features from situations, expressing these in words and symbols. They test propositions, and use formal mathematical arguments to test their truth, modifying them as required.

Students choose, use and develop mathematical models and procedures with attention to assumptions and constraints (for example, they test the suitability of the results of data analysis in terms of the context being modelled).

They solve problems in a wide range of practical, theoretical and historical contexts and communicate the results of these investigations. They extend

Definitions of underlined terms are provided in the Glossary (page 38)

their problem solutions by generalising, or changing the initial constraints of a situation for further investigation.

Students use technology (for example, geometry software, graphics calculators, spreadsheets and computer algebra systems) to develop mathematical ideas and solve problems.

They describe the major features of mathematical structure, and use of logical argument in mathematical discourse and applications of mathematics.

National Statements of Learning

This learning focus statement, with the following elaboration, incorporates the Year 9 National Statement of Learning for Mathematics.

Standards

Number

At Level 6, students comprehend the set of real numbers containing natural, integer, rational and irrational numbers. They represent rational numbers in both fractional and decimal (terminating and infinite recurring) forms (for example, $1\frac{4}{25} = 1.16$, $0.\overline{47} = \frac{47}{99}$). They comprehend that irrational numbers have an infinite non-terminating decimal form. They specify decimal rational approximations for square roots of primes, rational numbers that are not perfect squares, the golden ratio φ, and simple fractions of π correct to a required decimal place accuracy.

Students use the Euclidean division algorithm to find the greatest common divisor (highest common factor) of two natural numbers (for example, the greatest common divisor of 1071 and 1029 is 21 since 1071 = 1029 × 1 + 42, 1029 = 42 × 24 + 21 and 42 = 21 × 2 + 0).

Students carry out arithmetic computations involving natural numbers, integers and finite decimals using mental and/or written algorithms (one- or two-digit divisors in the case of division). They perform computations involving very large or very small numbers in scientific notation (for example, $0.0045 \times 0.000028 = 4.5 \times 10^{-3} \times 2.8 \times 10^{-5} = 1.26 \times 10^{-7}$).

They carry out exact arithmetic computations involving fractions and irrational numbers such as square roots (for example, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{(\frac{3}{2})} = \frac{\sqrt{6}}{2}$ and multiples and fractions of π (for example $\pi + \frac{\neq}{4} = \frac{5\neq}{4}$). They use appropriate estimates to evaluate the reasonableness of the results of calculations involving rational and irrational numbers, and the decimal approximations for them. They carry out computations to a required accuracy in terms of decimal places and/or significant figures.

Definitions of underlined terms are provided in the Glossary (page 38)

Space

At Level 6, students represent two- and three-dimensional shapes using lines, curves, polygons and circles. They make representations using perspective, isometric drawings, nets and computer-generated images. They recognise and describe boundaries, surfaces and interiors of common plane and three-dimensional shapes, including cylinders, spheres, cones, prisms and polyhedra. They recognise the features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use associated angle properties.

Students explore the properties of spheres.

Students use the conditions for shapes to be congruent or similar. They apply isometric and similarity transformations of geometric shapes in the plane. They identify points that are invariant under a given transformation (for example, the point (2, 0) is invariant under reflection in the x-axis, so the x axis intercept of the graph of $y = 2x – 4$ is also invariant under this transformation). They determine the effect of changing the scale of one characteristic of two- and three-dimensional shapes (for example, side length, area, volume and angle measure) on related characteristics.

They use latitude and longitude to locate places on the Earth's surface and measure distances between places using great circles.

Students describe and use the connections between objects/location/events according to defined relationships (networks).

Measurement, chance and data

At Level 6, students estimate and measure length, area, surface area, mass, volume, capacity and angle. They select and use appropriate units, converting between units as required. They calculate constant rates such as the density of substances (that is, mass in relation to volume), concentration of fluids, average speed and pollution levels in the atmosphere. Students decide on acceptable or tolerable levels of error in a given situation. They interpret and use mensuration formulas for calculating the perimeter, surface area and volume of familiar two- and three-dimensional shapes and simple composites of these shapes. Students use pythagoras theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles.

They use degrees and radians as units of measurement for angles and convert between units of measurement as appropriate.

Students estimate probabilities based on data (experiments, surveys, samples, simulations) and assign and justify subjective probabilities in familiar situations. They list event spaces (for combinations of up to three events) by lists, grids, tree diagrams, venn diagrams and karnaugh maps (two-way tables). They calculate probabilities for complementary, mutually exclusive, and compound events (defined using *and*, *or* and *not*). They classify events as dependent or independent.

Students comprehend the difference between a population and a sample. They generate data using surveys, experiments and sampling procedures. They calculate summary statistics for centrality (mode, median and mean), spread (box plot, inter-quartile range, outliers) and association (by-eye estimation of the line of best fit from a scatter plot). They distinguish informally between association and causal relationship in bi-variate data, and make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables.

Structure

At Level 6, students classify and describe the properties of the real number system and the subsets of rational and irrational numbers. They identify subsets of these as discrete or continuous, finite or infinite and provide examples of their elements and apply these to functions and relations and the solution of related equations.

Student express relations between sets using membership, $\in$, complement, ′ , intersection, $\cap$, union, $\cup$, and subset, $\subseteq$, for up to three sets. They represent a universal set as the disjoint union of intersections of up to three sets and their complements, and illustrate this using a tree diagram, venn diagram or karnaugh map.

Students form and test mathematical conjectures; for example, 'What relationship holds between the lengths of the three sides of a triangle?'

They use irrational numbers such as, π , φ and common surds in calculations in both exact and approximate form.

Students apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computation with number, to rearrange formulas, rearrange and simplify algebraic expressions involving real variables. They verify the equivalence or otherwise of algebraic expressions (linear, square, cube, exponent, and reciprocal, (for example,

$4x - 8 = 2(2x - 4) = 4(x - 2)$; $(2a - 3)^2 = 4a^2 - 12a + 9$; $(3w)^3 = 27w^3$;
$\frac{(x^3y)}{xy^2} = x^2y^{-1}$; $\frac{4}{xy} = \frac{2}{x} \times \frac{2}{y}$).

Students identify and represent linear, quadratic and exponential functions by table, rule and graph (all four quadrants of the cartesian coordinate system) with consideration of independent and dependent variables, domain and range. They distinguish between these types of functions by testing for constant first difference, constant second difference or constant ratio between consecutive terms (for example, to distinguish between the functions described by the sets of ordered pairs {(1, 2), (2, 4), (3, 6), (4, 8) …}; {(1, 2), (2, 4), (3, 8), (4, 14) …}; and {(1, 2), (2, 4), (3, 8), (4, 16) …}). They use and interpret the functions in modelling a range of contexts.

They recognise and explain the roles of the relevant constants in the relationships $f(x) = ax + c$, with reference to gradient and y axis intercept, $f(x) = a(x + b)^2 + c$ and $f(x) = ca^x$.

They solve equations of the form $f(x) = k$, where k is a real constant (for example, $x(x + 5) = 100$) and simultaneous linear equations in two variables (for example, $\{2x - 3y = -4$ and $5x + 6y = 27\}$ using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods.

Working mathematically

At Level 6, students formulate and test conjectures, generalisations and arguments in natural language and symbolic form (for example, 'if m^2 is even then m is even, and if m^2 is odd then m is odd'). They follow formal mathematical arguments for the truth of propositions (for example, 'the sum of three consecutive natural numbers is divisible by 3').

Students choose, use and develop mathematical models and procedures to investigate and solve problems set in a wide range of practical, theoretical and historical contexts (for example, exact and approximate measurement formulas for the volumes of various three dimensional objects such as truncated pyramids). They generalise from one situation to another, and investigate it further by changing the initial constraints or other boundary conditions. They judge the reasonableness of their results based on the context under consideration.

They select and use technology in various combinations to assist in mathematical inquiry, to manipulate and represent data, to analyse functions and carry out symbolic manipulation. They use geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

Glossary

algebra

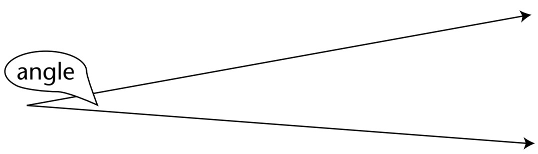
Refers to both the process of *manipulating* variables and constants in a mathematical expression according to fixed laws, properties or rules; for example, simplifying an expression or solving an equation; and alternatively to a *mathematical structure* whose elements and operations satisfy a given *collection of laws*. The word algebra comes from the work of the Arabic scholar *Abu Abd-Allah ibn Musa al'Khwarizmi*, who was born about 790 AD near Baghdad, and died about 850 AD. Khwarizmi wrote one of the first books *Hisab al-jabr w'al-muqabala* on what is now called algebra in 830 AD. *Al-jabr* refers to the process of moving a subtracted quantity to the other side of an equation while *al-muqabala* involves subtracting equal quantities from both sides of an equation. In 1140 AD this text was translated into Latin as *Liber algebrae et almucabala*, from which the word algebra has become part of mathematical language.

algorithm

A process for computation that can be carried out mechanically; for example, the algorithm for subtraction of many-digit decimal numbers, or the algorithm for factorisation of a linear expression using the distributive rule. The word *algorithm* comes from the old English *algorisme*, from a Latin translation of the name of the ninth century AD Arabic scholar *al-Khwarizmi*, who investigated computation using the Hindu numeration system (leading to the Hindu-Arabic number system of today).

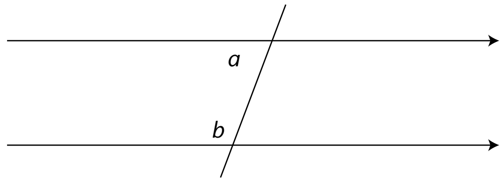
angle

An angle is formed at the point of intersection of two rays:

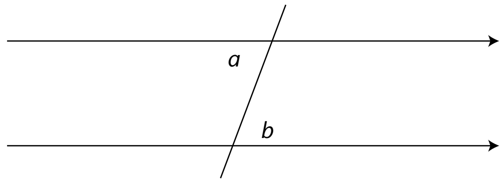


Angle *measure* is commonly based on the amount of turn between two rays with a common point. There are three common measures of angle: *fraction of a full turn*, *degree* and *radian*.
A full turn = 360 degrees = 2π radian.
A half turn = 180 degrees = π radian.

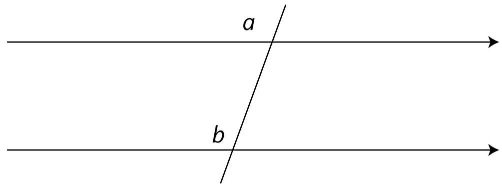
- **allied or co-interior (angles)**: Angles which are between two parallel lines, adjacent to a transversal cutting that pair of parallel lines and sum to 180 degrees are said to be *allied* or *co-interior* angles, for example angles *a* and *b* in the following diagram:



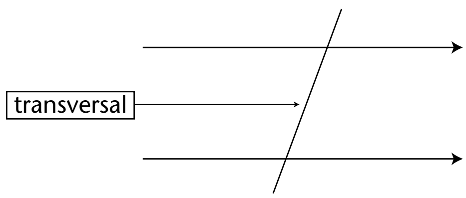
- **alternate (angles)**: Angles which are symmetrically located under a half turn with respect to the midpoint of a transversal cutting a pair of parallel lines are said to be *alternate* angles, for example angles *a* and *b* in the following diagram:



- **corresponding (angles)**: Angles which are in the same relative position with respect to a transversal cutting a pair of parallel lines are said to be corresponding angles, for example angles *a* and *b* in the following diagram:



- **supplementary (angles)**: Angles at a point on a line that sum to 180 degrees are said to be supplementary angles.
- **transversal**: A line that cuts a pair of parallel line obliquely



approximate

To obtain a value to a particular accuracy. For example, the fraction $\frac{22}{7}$ provides an approximate value for the irrational real number π. Rounded correct to 4 decimal places, $\frac{22}{7}$ has the value 3.1429 while π has the value 3.1416. These values are themselves decimal approximations to $\frac{22}{7}$ and π

respectively. While it is not possible to trisect any angle *exactly* using only a compass and rules, it is possible to approximate such a trisection with reasonable accuracy.

associative

An operation is *associative* if the result of applying the operation to any three elements of an expression is the same regardless of which pair of elements (without changing their order) is combined first.

Addition and multiplication *are* associative on the set of natural numbers, for example:

4 + (7 + 5) = 4 + 12 = 16 and (4 + 7) + 5 = 11 + 5 = 16

2 × (3 × 4) = 2 × 12 = 24 and (2 × 3) × 4 = 6 × 4 = 24

Subtraction and division are *not* associative on the set of natural numbers, for example:

10 – (4 – 2) = 10 – 2 = 8 but
(10 – 4) – 2 = 6 – 2 = 4

24 ÷ (12 ÷ 2) = 24 ÷ 6 = 4 but
(24 ÷ 12) ÷ 2 = 2 ÷ 2 = 1

In general the *associative* laws (properties) for addition and multiplication of real numbers state respectively that *for all* real numbers a, b and c:

$a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$

assumption

A proposition which is taken as being true with respect to a given context. For example, in a modelling problem to design a seating arrangement in a theatre, it may be assumed that the height of the people watching the movie is no greater than 200 cm.

bi-variate (data)

Data relating to the simultaneous measurement of two variables; for example, age and income.

chance and likelihood

The relative frequency of an event, this may be expressed qualitatively using terms such as: impossible, no chance, not likely, an even change, odds-on, likely, a certainty, or quantitatively using numbers on a scale from 0 (impossible) to 1 (certain). These numerical values are often expressed as fractions such as $\frac{1}{2}$, ratios such as 2:3, decimals such as 0.87 or percentages such as 40%.

closure

The result of carrying out an operation on an element of a set, or elements of a set, is also an element of that set. For example, multiplication is closed on the set of natural numbers, because the result of multiplying any pair of natural numbers is also a natural number. Division is *not* closed on natural numbers, since 9 and 2 are both natural numbers, but the result of dividing 9 by 2 is not a natural number: $9 \div 2 = \frac{9}{2} = 4.5$, and 4.5 is a decimal fraction, not a natural number.

commutative

An operation is *commutative* if the result of applying the operation to any two elements of a set is the same, regardless of the order of the elements. Addition and multiplication *are* commutative on the set of natural numbers, for example:

6 + 12 = 18 = 12 + 6 and 6 × 12 = 72 = 12 × 6

but subtraction and division are not commutative for example:

6 – 12 = – 6 but 12 – 6 = 6 and $6 \div 12 = \frac{1}{2}$ but
12 ÷ 6 = 2.

In general the *commutative* laws (properties) for addition and multiplication of real numbers state that *for all* real numbers a and b, $a + b = b + a$ and $ab = ba$, respectively.

complement (set)

The set of all elements *not* in a given set with respect to the universal set for a particular context or situation. For example, if the universal set in a particular situation is taken to be the letters of the alphabet, the complement to the set of vowels is the rest of the alphabet. If the universal set in a particular situation is taken to be the set of numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then the complement to A = {4, 5, 6} is {1, 2, 3, 7, 8, 9, 10}. The complement of A is written as A' or $\overline{A}$.

composite number

A natural number which has *more than two distinct elements* in its factor set; for example, 8 is a composite number as it has four distinct elements in its factor set: {1, 2, 4, 8}. The number 2 is not a composite number since it has only two distinct elements in its factor set: {1, 2}. With the exception of 1, which has only one distinct element in its factor set: {1}, all natural numbers are either *composite* or *prime*.

congruence

The property of being *identical* in shape and dimensions. Two shapes are congruent if one of them can be mapped onto the other by transformations that do not change the length of line segments or the angle between lines.

conjecture

A statement whose truth or otherwise is not yet determined, but is open to further investigation, for example, *Golbach's Conjecture: "every even natural number greater than 2 can be expressed as a sum of two prime numbers"*. First stated in 1742, the Golbach conjecture has not yet been either proven to be true or shown to be false, although many mathematicians intuitively believe that it is true.

connected
Two points in the plane are said to be connected if there is a line or curve (edge) that joins them. A set of points in the plane, such as a *network*, is said to be connected if there are no two points in the set which are not connected.

connective
A logical term that connects or qualifies other expressions, such as 'and', 'or', 'not', 'if ... then ...' and 'is equivalent to'. For example, given a set of attribute blocks, specifying the blocks that are red and square involves two attributes 'red', 'square' which apply to some blocks but not to others. The use of the connective *and* to specify 'red' *and* 'square' required both attributes to apply.

constant
A number that has a fixed value in a given context. For example, in the calculation of $n + 11$ for different natural numbers n, the number 11 is a *constant*. In formulas such as $P = 4 \times l$, 4 is a *constant* while P and l are *variables*.

constraint
A condition which is applied in a given context; for example, in solving the equation $3x + 2y = 24$ a *constraint* may be that only natural number solutions are required (there are an infinite number of integer solutions).

continuous
Can in principle assume all possible values in a given interval. For example, height is a continuous data measurement. While the actual height of a person can only be physically measured to a given accuracy, it is possible in principle for a for person's height to be any value within a typical range of heights for a human being.

correspondences

- **many-to-one correspondence**: A function between two sets where each element in one set (domain) corresponds to exactly one element in the other set (range); however, an element in the range may be mapped onto by more than one element in the domain. For example, each student in a class has exactly one height measure (to the nearest centimetre) at a given instant (so the relation 'the height of' is a function) but it may be the case that two students are the same height.
- **one-to-one correspondence**: A function between two sets where each element in one set (domain) corresponds to exactly one element in the other set (range) and vice versa. Thus, in a ballroom dancing class, there will be a one-to-one correspondence between male and female partners during a given dance.
- **range (of a relation)**: A relation is a correspondence between two sets. When one of the sets is specified as the set of elements *to* which the correspondence is made, this set is called the *co-domain* of the relation. The *subset* of the *co-domain* to which elements of the domain are actually matched up, is called the *range* of the relation. For example, if a correspondence is formed between the *students* in a class and their favourite colour, the *students* would be a natural choice for the *domain* of relation. The *range* corresponds to the actual set of favourite colours of the students. The *co-domain* is the set of all possible colours, not just the favourites of the class of students. Thus, the *range* of a relation is, in general, a *subset* of its *co-domain*.

counter-example
An *instance* where a proposition or conjecture is false. The number 6 is a counter-example to the propostion that every even number is also a multiple of four.

counting
The process of listing a subset of the set of natural numbers $N = \{0, 1, 2, 3 ... \}$ in consecutive order; for example $\{0, 1, 2, ...\}$ or $\{11, 12, 13, ...\}$.

decimal number
A number expressed using the base 10 place value system. For example 412.56 is a decimal number.

degree
Measure of angle where a full rotation around a fixed point corresponds to 360 degrees.

discrete
Can only assume a countable number of data values. For example, shoe size is a discrete data measurement.

distributive
An operation is said to be distributive over another operation if it can take priority over the operation used for combination within brackets (that is, its application can be distributed over the brackets). Multiplication is distributive over addition for real numbers, for example:

$6 \times 17 = 6 \times (10 + 7) = (6 \times 10) + (6 \times 7) = 60 + 42 = 102$

The distributive property underpins algorithms for multiplication and division that involve natural numbers of several digits. In general the distributive law (property) for (multiplication over addition) for real numbers states that for all real numbers *a*, *b* and *c*:

$a(b + c) = ab + ac$.

Addition is *not* distributive over multiplication, for example:

$3 + (2 \times 4) = 3 + 8 = 11$ but
$(3 + 2) \times (3 + 4) = 5 \times 7 = 35$

division

For a finite set this is the process of partitioning the set into subsets of equal size. For natural numbers division re-expresses a given natural number in terms of a multiple of a smaller natural number and a remainder. For example $68 = 7 \times 9 + 5$, so 68 *divided by* 9 is equal to 7 with 5 remainder. Using rational numbers in fraction form this is expressed exactly as:

$$68 \div 9 = \frac{68}{9} = 7\frac{5}{9}$$

In general, for real numbers, if $xy = z$ then

$z \div y = \frac{z}{y} = x$, unless $y = 0$ in which case the process is not defined.

- **partition**: To divide into separate parts which together constitute the whole. For example, the letters of the alphabet can be partitioned into vowels and consonants, the set of natural numbers can be partitioned into those with remainder 0, 1 or 2 on division by 3.

domain

A relation or function is a map between the elements of two sets. The set *from* which the mapping occurs is called the *domain* of the function or relation. The set onto which the elements of the domain are mapped is called the *co-domain*. For example, the domain of a relation (the favourite colour of) could be the students in a class, while the co-domain is a set of colours. Not all the colours may be selected as a favourite for some student in the class. The *range* of the relation is a subset of the co-domain, which corresponds to the set of favourite colours for that class.

edge

A straight line or curve that forms the boundary of a region in the plane (such as the side of a triangle, or an edge in a network) or a boundary between two surfaces (such as the rim of a can or the edge of a box).

- **adjacency**: Two edges in a shape are said to be adjacent if they meet at a common vertex; similarly, two faces in a shape are said to be adjacent if they meet at a common edge.

empirical

Derived from observation, measurement or experiment.

enlargement

One shape is an enlargement of another shape if they are similar and the scale factor for dilation is greater than 1. The dilation transformation involved is also referred to as an enlargement.

equation

A mathematical expression that includes the '=' symbol. Equations are used to assign a value to a pro-numeral; for example, $a = 2$; to define the rule of a function; for example $y = 2x + 3$, where whatever value *x* takes, *y* is two times *x* plus three; and to specify conditions that must be satisfied by the value of a variable; for example, if $2x + 3 = 10$, then $x = 4$ for this statement to be *true*.

equivalence

Two statements or propositions are understood to be equivalent if they are *both* true or *both* false. That is, the conditions which make one true make the other true as well, and the conditions which make one false make the other false as well.

equivalent fraction

Given any fraction, an equivalent fraction is one whose numerator and denominator is a common integer multiple of the numerator and denominator of the given fraction. For example, an equivalent fraction of $\frac{1}{2}$ is $\frac{2}{4}$, with a common integer multiple of 2 such that $\frac{(1\times2)}{(2\times2)} = \frac{2}{4}$ For each fraction expressed in simplest form an equivalence class (or family of equivalent fractions) can be generated by successively multiplying its numerator and denominator by the natural numbers (excluding zero). For example, for the fraction $\frac{2}{3}$, the corresponding family is:

$$\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, ...\}$$

error

Is the difference between an actual value and its measured or estimated value and is defined as:

error = measured or estimated value – actual value

estimate

To form an approximate value for a quantity.

example

An *instance* where a proposition or conjecture is *true*. The number 6 provides an example of a number which is even and a multiple of three.

extrapolation

Working beyond known data to make predictions; for example, working past the last known point on a graph to predict a value beyond this point.

face/s

A bounded surface: a bounded region in a network, or on a three-dimensional shape or object.

- **adjacency**: Two edges in a shape are said to be adjacent if they meet at a common vertex; similarly, two faces in a shape are said to be adjacent if they meet at a common edge.

factor (number)

A natural number that divides exactly into a given natural number. For example, 2 is a factor of 12, since 2 × 6 = 12. The set of all factors of a given number is called its *factor set*. The *factor set* of 12 is {1, 2, 3, 4, 6, 12}. The elements of a factor set are often grouped in pairs. Thus, the set of *factor pairs* of 12 is {{1, 12}, {2, 6}, {3, 4}, {4, 3}, {6, 2}, {12, 1}}. This concept also applies to algebra, where, for example, the factors of the linear expression $6x + 9$ are 3 and $2x + 3$ since $3(2x + 3) = 6x + 9$.

factorial

The number formed by the product of a given natural number with all the natural numbers less than it down to 1. For example 4 *factorial* is $4! = 4 \times 3 \times 2 \times 1 = 24$.

finite

The set {*a*, *b*, *c*, *d*, *e*} is an example of a finite set. The set of all people alive on a given day is a very large, but finite set. The cardinal number of a finite set is a natural number, that is, the elements of any finite set can be put in a one-to-one correspondence with the elements of a set of the form {0, 1, 2, 3, … , *n*} where *n* is a natural number.

formal unit

A unit whose value is fixed by agreement; for example, litre is a formal unit of capacity for fluids and hour is a formal unit of time.

fraction

See also *rational number*. In a fraction, for example $\frac{3}{4}$, the number 4 is called the *denominator* of the fraction (it specifies the number of equal sized partitions of a whole unit), the number 3 is called the *numerator* (this specifies how many of these parts), and the horizontal line indicating the part-whole relation of the fraction is called the *vinculum* (from the Latin meaning a line or stroke that connect quantities). A fraction is said to be expressed in *simplest form* if its numerator and denominator have no common factor, that is are co-prime. For example, $\frac{3}{4}$ is expressed in simplest form, but $\frac{6}{12}$ is not in simplest form, since 2 is a common factor of both 6 and 12 (as are 3 and 6).

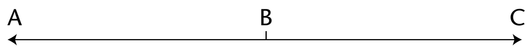
function

A correspondence (map or relation) between the elements of two sets where each element in the first set is mapped to exactly one corresponding element in the second set. A function is either a one-to-one correspondence or a many-to-one correspondence.

- **dependent variable**: The variable associated with the *range* of a relation. For a function f with rule $y = f(x)$, y is the dependent variable.
- **independent variable**: The variable associated with the *domain* of a relation. For a function f with rule $y = f(x)$, x is the *independent* variable.

golden ratio (phi φ)

Is the irrational number whose value is given by the proportion AC : AB = AB : BC where A and C are the endpoints of a line segment and B is the point on the line segment between A and C such that AC : AB = AB : BC



It is called the golden ratio as it is believed to represent a proportion of lengths that is aesthetically attractive to the human eye in art and design contexts. The exact value of φ is $(1+\sqrt{5})/2$ and its approximate value is 1.618 correct to 3 decimal places. The decimal expansion for *phi* to 100 significant figures is:

1.618033988749894848204586834365638117720309179805762862135448622705260462818902449707207204189391137.

The digits in this decimal expansion do not display any recurring pattern, a property which distinguishes irrational numbers from rational numbers.

graph

A visual representation of data or functions. Cartesian graphs of functions and relations are plots of ordered pairs of values (x, y) that represent the function or relation relative to x and y coordinate axes and the fixed origin (0, 0). Statistical graphs include dot plots, box and whisker plots, bar graphs and histograms.

highest common factor (hcf)

Also called the *greatest common divisor* (*gcd*). Given any two natural numbers, this is the largest natural number that divides both of them exactly, that is, the highest common number in their factor sets. For example, the *hcf (gcd)* of 18 and 24 is 6. Any fraction can be expressed in simplest terms by dividing its numerator and denominator by their *hcf (gcd)*.

identity

An element of a set which when combined (using a given operation) with any other element of the set leaves that element unchanged. For example, 0 is the identity element for addition of natural numbers, since for any natural number n it is the case that $0 + n = n$ and $n + 0 = n$. Similarly, 1 is the identity element for multiplication of natural numbers, since for any natural number n it is the case that $1 \times n = n$ and $n \times 1 = n$.

implication

A statement of the form *if … then …* An implication is understood to be *true* unless the *first* part of the statement is *true* but the second part of the statement is *false*.

inclusion (subset)

A set A is a subset of another set B if all of the elements of A are also elements of B. For example, if A = {vowels} and B = {letters of the alphabet} then A is a (proper) subset of B, written symbolically as $A \subset B$. In the case where A is required to be a subset of B, but may include all of the elements of B then this is represented symbolically by $A \subseteq B$.

inequality

A mathematical expression containing the terms 'less than', 'less than or equal to', 'greater than', or 'greater than or equal to' their respective symbolic representations '<', '≤', '>' and '≥'. For example 'the set of prime numbers less than or equal to 29', is an inequality as is the expression $2y \geq x^2$ where x and y are real numbers.

inference

An assertion made on the basis of analysis from given data or propositions; for example, on the basis of the weather patterns observed over several years, a farmer might infer that it is likely to be a hot summer.

infinite

The set of natural numbers $N = \{0, 1, 2, 3 \ldots\}$ is an example of an infinite set. There are many examples of infinite sets, the set of all prime numbers is an infinite set (there is no largest prime number). The set of natural numbers, N , is an example of an infinite set which has a smallest element, 0, but no largest element. The set of integers $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$ is an example of an infinite set which has no smallest or largest element. The set {0.9, 0.99, 0.999, 0.9999, … , 1} is an example of an infinite set which has both a smallest element, 0.9, and a largest element, 1. It is *not* possible for the elements of any infinite set to be put in a one-to-one correspondence with the elements of a set of the form $\{0, 1, 2, 3, \ldots , n\}$ where n is a natural number.

informal unit

A unit whose value is decided on in a given context, for example, the use of a pace to measure distance or the use of a cupped hand to measure capacity of rice for a meal (irregular informal units). An informal unit may also be regular, such as the use of paperclips to measure length or a drinking glass to measure a small amount of a substance (capacity).

integer

An element of the infinite set of numbers $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$, sometimes also referred to as a positive or negative whole number.

interpolation

Working within known data to make predictions, for example working between two known points on a graph to predict a value in between these points.

intersection (set)

Given two sets A and B, their intersection, written $A \cap B$, is the set of all elements common to both sets. If A and B have no elements in common then their intersection is the empty set { }. For example, if $A = \{a, b, d, z\}$ and $B = \{a, c, x, y, z\}$ then $A \cap B = \{a, z\}$; however, if $C = \{m, n\}$ then $A \cap C = \{\ \}$.

invariance

The property of not changing under a process such as transformation; for example, the points on a mirror line are invariant under the transformation of reflection in that mirror line. If a person touches a mirror with their finger, then the point of contact will be invariant under reflection in the mirror, all other points their image will have left- and right-hand senses reversed.

inverse

For each element of a set its inverse with respect to a given operation defined on the set is the element in the set which, when they are combined using the operation result in the identity element. For example, the inverse of the integer +4 with respect to the operation of addition is the integer –4 since $+4 + (-4) = 0$ and $-4 + (+4) = 0$. The inverse of the rational number $\frac{2}{3}$ with respect to the operation of multiplication is the rational number $\frac{3}{2}$ since $\frac{2}{3} \times \frac{3}{2} = \frac{1}{1} = 1$.

investigation
Exploration of a situation or context.

irrational number
A number that cannot be expressed as a fraction in the form $\frac{m}{n}$, where *m* and *n* are integers and *n* is non-zero. The decimal form of such numbers does not terminate, and is non-recurring, that is, there is no finite sequence of digits that repeats itself. For example, $r = 0.12345678910111213$ is part of the decimal expansion of an irrational real number. Numbers such as √2, the golden ratio φ, and π are examples of irrational numbers.

karnaugh map
A diagram consisting of a small number of non-overlapping (mutually exclusive) rectangles used to indicate the relationship between elements of a set and given properties or attributes.

location
A description of position with respect to some fixed reference.

logic
Principles of reasoning where one proposition is deduced from other propositions.

lowest common multiple (lcm)
Given any two natural numbers, their lowest common multiple is the smallest natural number which they both divide exactly. This is *not* necessarily their product. For example, the *lcm* of 6 and 9 is 18, since 3 × 6 = 18 and 2 × 9 = 18, but 6 × 9 = 54. The *lcm* is used in the operation of addition and subtraction of fractions, to identify the equivalent fractions with the same denominator.

magnitude
The size, or absolute value of a number; for example, both +5 and –5 have magnitude 5. The magnitude of certain numbers can only be approximated to a given accuracy, for example the magnitude of the number *pi*, correct to two decimal places, is 3.14.

mean
The sum of values in a data set divided by the total number of values in the data set. For example, is a data set consists of the values $\{x_1, x_2, x_3, \ldots, x_n\}$, then the mean is defined as:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

measure
A measure is a record of the magnitude of an attribute (such as weight, length, time, and likelihood) associated with an object or event.

measure of centre
Also called an *average*. This is a statistic that is used to represent a data set. There are three common measure of centre for a data set: *mode* (the most common value), *median* (the middle value) and the *mean* (the sum of all the values divided by the number of values).

median
The middle value of a data set when its elements are arranged in numerical order. For an even set of elements the mode is taken to be the value half way between the two middle values.

mensuration formulas
Define the measure of one quantity as a function of other quantities using an algebraic formula. For example, the area, *A*, of a circle radius, *r*, is defined by the mensuration formula $A = \pi r^2$and the average speed, *s*, of a moving object which travels a distance *d* in time *t* is defined by the formula $s = d/t$.

mode
The *most common* value in a data set. If there are two such common values the data set is sometimes said to be bi-modal. In some cases it is not useful to consider the mode as a measure of centre for a data set.

modelling
Using mathematical concepts, structures and relationships to describe and characterise, or model, a situation in a way that captures its essential features.

natural number
An element of the infinite set of numbers *N* = {0, 1, 2, 3 ...}, sometimes also referred to as a counting number. In some references the number 0 is not included in the set of natural numbers; however, it is useful to include as it corresponds to the number of elements in an empty set.

- **even natural number**: An element of the set {0, 2, 4, 6, ...}
- **odd natural number**: An element of the set {1, 3, 5, 7 ...}

negative number
A real number *x* is negative if $x < 0$. The set of negative integers (regular whole numbers) $Z^- = \{-1, -2, -3, \ldots\}$ is sometimes referred to as 'the negative counting numbers'.

net

A two-dimensional representation of a three-dimensional shape in such a way that it can be folded to construct the three dimensional shape.

network

A set of points (*vertices* or *nodes*) some of which are joined by lines or curves (*edges*) which sometimes enclose regions (*faces*). Networks are used to represent relationships involving connectedness; for example, road networks, a family tree or the edges lining a tennis court.

numeral

The *designation* of a number in a given language; for example, the number 'three' is designated by the Hindu-Arabic numeral **3**, the Roman numeral **III**, and the Chinese numeral 三.

order

Is a relation that describes the location of elements in a set with respect to each other. These elements may be totally ordered or partially ordered. For example, the set of natural numbers is totally ordered by the relation less than or equal to since for any two natural numbers m and n exactly one of the following is true: $m < n$ or $m = n$ or $m > n$. Similarly, the set of students in a class can be totally ordered with respect to their height using the relation less than or equal to. However, the set of people at a school fair is only partially ordered by the relation 'is a parent of' since there will likely be many pairs of people who are not each others parent, such as siblings.

ordered pair

A special type of set of two elements for which order is significant. For example, the grid reference used on a map is an ordered pair such as (K, 7) where K is the horizontal grid reference on the directory page and 7 is the vertical grid reference on the directory page.

percentage

A ratio expressed as a proportion to 100.

perimeter

The boundary of a closed shape or curve, also the *length* of this boundary.

periodic

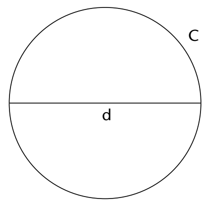
Repeats at regular intervals; for example, if the elements of the set of natural numbers are divided by 3, in order, and the remainder on division recorded, the following periodic pattern of remainders occurs:

0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2 ...

The graph of a function that describes the rise and fall of tides in a given location from high tide to low tide over several days will also be periodic.

pi

Represented by the symbol π, is the irrational number defined by the ratio of the circumference C of a circle to its diameter, d:



Its approximate value, correct to 2 decimal places is 3.14, and $\frac{22}{7}$ is a reasonably accurate fraction approximation to π. The decimal expansion for *pi* to 100 significant figures is:

3.141592653589793238462643383279502884197
16939937510582097494459230781640628620899
86280348253421170 68.

The digits in the continued decimal expansion of π do not have any recurring pattern, a property which distinguishes irrational numbers from rational numbers.

platonic solids

The five platonic solids are the *tetrahedron* (4 equilateral triangles as faces), the *cube* (six squares as faces), the *octahedron* (8 equilateral triangles as faces), the *dodecahedron* (12 regular pentagons as faces), and the *icosahedron* (20 equilateral triangles as faces). They are solid shapes with faces that are made of regular polygons which tessellate with an equal number of faces at each vertex.

polygon

Literally 'many-sides'; a closed plane figure with sides formed by straight lines; for example, triangles, quadrilateral, pentagons, hexagons and the like. Polygons with all sides of equal length and all angles between adjacent sides equal are said to be *regular* polygons.

- **tangram**: A Chinese puzzle formed by a square cut into several pieces that are then rearranged to create other shapes.

polyhedron

A three-dimensional shape whose faces are adjacent polygons; for example, a pyramid is a polyhedron but a cone is not a polyhedron (part of a cone is a curved surface which is not a polygon).

population

The complete or universal set for a given context or situation. This may refer to the Australian population with respect to an election, or the population of wombats in Victoria.

power (exponent or index)

The number $4^3 = 4 \times 4 \times 4 = 64$ is read as four to the power (also called the *exponent* or *index*) 3. A function f of the variable x, with rule of the form $f(x) = a^x$ where a is a fixed constant and the variable x is the *exponent* is called an *exponential* function.

prime number

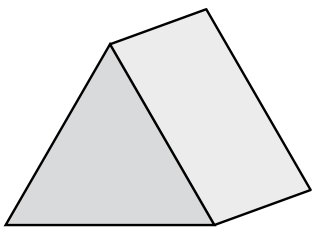
A natural number that has *exactly two distinct factors*, 1 and itself. The number 1 is not a prime number (it has only one distinct factor), nor is the number 8, as it has four distinct factors {1, 2, 4, 8}. The number 2 is the only even prime number. The set of the first 100 prime numbers is:

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541}

There is no known function for generating the sequence of prime numbers, although there are algorithms for identifying whether a number is prime or not.

prism

A three-dimensional shape that has a polygonal cross section, formed by having its edge points translated parallel to a given direction. For example, the following shape is a *triangular prism*:



problem posing

Formulating a problem in such a way that it is amenable to mathematical analysis.

problem solving

The application of mathematical reasoning to the development of a solution or solutions to a given problem.

proportion

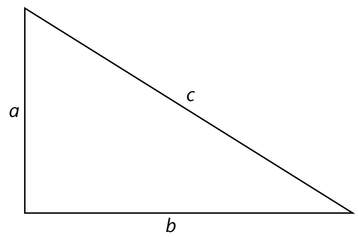
Two ratios are said to be in proportion if they are equivalent ratios. That is a:b is in proportion to c:d if there is a non-zero real number k such that a:b = kc:kd.

pyramid

A three-dimensional shape that has a square base and edges from the corners of the base to a point (the *vertex*) above the square base. If the vertex is vertically above the centre of the square base, the pyramid is said to be a *right* pyramid (the line segment connecting the centre of the square base to the vertex is at 90 degrees – a *right* angle – to the plane of the square base).

pythagorean relationship (pythagoras theorem)

If a, b and c are the lengths of the sides of a right angled triangle, such that c is the length of the side opposite the right angle, then $a^2 + b^2 = c^2$:



radian

Measure of angle where one cycle around the circumference of a unit circle from a fixed point corresponds to 2 π radians. Thus, as a measure of angle, 180 degrees is equivalent to π radian.

random

Not able to be predicted, not regular, a chance event.

random number

A number which is generated at random. The nth digit of the decimal expansion of a rational number is *not* random since these numbers can be predicted in advance. Selecting an arbitrary sequence of digits in the decimal expansion of an irrational real number can be used as a random number generator since the decimal expansion have no repetition. For example, the decimal expansion of *pi* could be used for this purpose.

range

The difference between the largest value and the smallest value in a data set.

ratio

A comparison a:b of the size of two (or more) quantities relative to each other.

rational number

An element of the infinite set of numbers:

$Q = \{\frac{m}{n}$, where m and n are integers and n is not equal to zero$\}$, sometimes also referred to as a fraction. Rational numbers can be expressed in fraction form, with corresponding terminating decimal expansion, for example, $\frac{1}{8} = 0.125$, or with infinite recurring decimal expansion, for example, $\frac{4}{9} = 0.444\ldots$

Rational numbers may be positive or negative; for example, $\frac{-17}{5}$ is also a rational number.

recursion

The process of carrying out the current step of a process using the results of the previous step (or steps) of the same process. For example, the sequence of numbers:

{3, 6, 12, 24… } can be described using recursion as 'start at 3 and make the next term in the sequence twice the previous term in the sequence'. Students often intuitively define sequences using recursion. Skip-counting; for example, 'counting by fives starting from 12' is another example of a recursive process.

reflection

A transformation where each point in the plane is reflected in a given mirror line.

relation

A correspondence (map) between the elements of two sets, for example, the relation 'has the favourite colour' between the set of students in a class (the *domain* of the relation) and the set of colours (the *co-domain* of the relation). A relation can be represented as a set of ordered pairs, a map (arrow diagram) or a graph.

remainder

If m and n are two natural numbers with m greater than n, such that $m = p \times n + r$ where p and r are also natural numbers, then r is said to be the remainder of m on division by n. For example, if $m = 29$ and $n = 6$, then $m = 4 \times n + 5$, so the remainder of 29 on division by 6 is 5.

rotation

A transformation where each point in the plane is rotated through a given angle about a fixed point (the point of rotation).

rounding

The process for *approximating* a value that lies between two known values by one of these values. In particular, when a measure lies between two of the smallest marks on a scale it is rounded to the nearest value represented by either one of these marks. Rounding is used to specify a number correct to a given accuracy. With *measurements* this often means rounding a decimal number. For example, 16.29 rounded to the nearest *tenth* of a unit is 16.3, rounded to the nearest *unit* is 16, rounded to the nearest *five* is 15, and rounded to the nearest *ten* is 20. The decimal number 57.139 rounded to the nearest *hundredth* is 57.14 and rounded to the nearest *tenth* is 57.1. However, 57.199 rounded to the nearest *hundredth* is 57.20. Some general principles for rounding are:

- if the last digit is a 1, 2, 3 or 4 then the previous digit is left unchanged and the number is said to be *rounded down*; for example, 296.2 rounded to the nearest *whole number* is 296
- if the last digit is a 6, 7, 8 or 9 then the previous digit is increased by 1 and the number is said to be *rounded up*; for example, 296.8 rounded to the nearest *whole number* is 297
- if the last digit is a 5 then the previous digit can be *randomly* rounded up or down, especially where several measurements are taken. This avoids cumulative error that would arise from either *always* rounding up or *always* rounding down; for example, 296.5 would be randomly rounded to either 296 or 297
- if rounding to a given accuracy has a cumulative effect, zeroes should be used to indicate the accuracy; for example, 299.97 rounded to the nearest *tenth* is 300.0.

It should be noted that several different conventions for rounding a last digit of 5 can be found in the literature, and these relate to different contexts for number computation.

sample

A subset of a population; for example, a set of people used for a newspaper survey is a sample of the population. A *random* sample is one which is obtained by using a random process for selecting a sample.

scale

Scale specifies the proportion between two measures. For example, a model of a house may be made on a 1:10 *scale* of length. A measuring *scale* for weight could be based on the extension of a spring in proportion to the mass of an object (each 500 grams could cause an extension 5 cm). The tick marks on the axes of a graph are specified according to some scale, for example each mark along the horizontal axis might correspond to 5 units, while each mark along the vertical axis might correspond to 2 units. These are then referred to as axes *scales*.

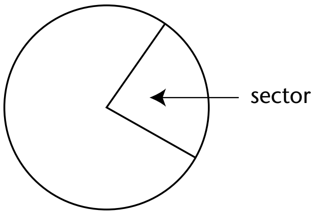
scientific notation (also standard form)

A notation used in particular to express very small or very large numbers in the form of the product of a decimal number between 1 and 10 (not inclusive) with an integer power of 10 expressed in exponential form. For example, in scientific notation:

1 567 000 = 1.567×10^6 and 0.000034 = 3.4×10^{-5}

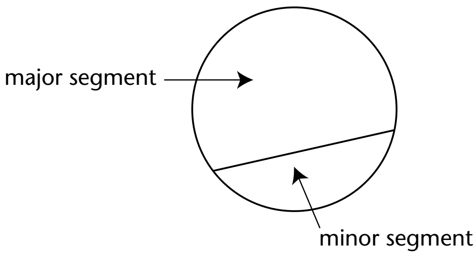
sector

The interior part of a circle formed by two radiuses:



segment

The interior part of a circle formed by a chord:



set

'set' is an undefined term that informally corresponds to the notion of a collection of objects or elements. Sets are usually specified by listing their elements, for example: { a, e, i, o, u }, describing them in words; for example, 'the set of Australian citizens', by or using a mathematical rule:

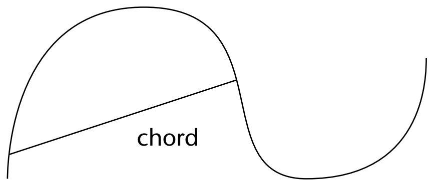
$\{(x, y): y = 2x + 1, x \in N\} = \{(0, 1), (1, 3), (2, 5), (3, 7) \ldots\}$.

- **element**: is an undefined term that informally corresponds to the notion of *belonging* or *membership* of a set. For example, 3 *is a member* of the set of natural numbers $N = \{0, 1, 2, 3, \ldots\}$. This relation can be written more concisely as $3 \in N$. The symbol '$\in$' is a short-hand for 'is an element of'. The number $\frac{1}{2}$ is not a natural number, and this can be written as $\frac{1}{2} \notin$ N, where is a shorthand for 'is **not** an element of'.
- **power set**: of a given set is the set of all possible subsets of the given set, including the empty set and the given set itself. For example, if $A = \{a, b, c\}$ then the power set of A, written $P(A)$ is the set $\{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. If there are n elements in the set A then there are 2^n elements in its power set, in this example A has 3 element and its power set has $2^3 = 8$ elements.
- **universal set**: In a given context, the *universal set* is the set of all objects under consideration. For example, the set of natural numbers is often the universal set for basic arithmetic computation in the early years of schooling. When students conduct a survey about students in their school, the set of all students in the school is the universal set (often called the population in this situation) for the survey.

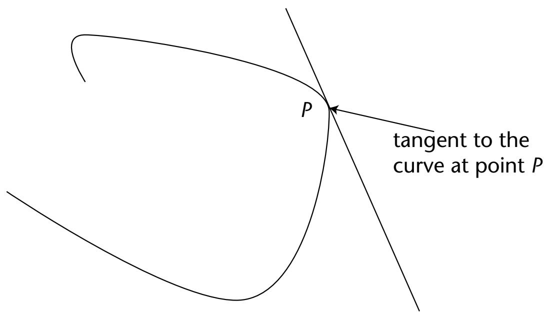
shape

A geometric object or representation of a common real-life object, in two-dimensional space, such as a free-hand closed curve, a triangle, circle, square; or in three-dimensional space (also called solids) such as a blob of play-dough, a cube, sphere or pyramid.

- **chord**: A line segment joining two points on a curve:



- **cross-section**: The shape produced by cutting a three-dimensional shape completely through by a plane.
- **shadow projection**: The two-dimensional image formed on a plane surface by the shadow of a three-dimensional object illuminated by a light source; for example, a person's shadow on the ground on a sunny day. In geometry this usually corresponds to the projections of a shape onto three plane surface at right angles to each other, such as front view, side view, top view.
- **tangent (local tangent)**: A line that touches but does not cut a curve at a point:



In the case of a circle, a tangent is always at right angles to the radius which meets the circumference of the circle at the point of contact.

significant figure

If a numerical value is expressed in scientific notation (standard form) $a \times 10^n$, where $1 < a <$ c and n is an integer, then all the digits in a are *significant*. For example, 1567000×10^6 has four significant figures and $0.000034 = 3.4 \times 10^{-5}$ has two significant figures.

similarity

The property of one shape being an exact enlargement or reduction of another shape.

simulation

The process of modelling an event using various devices or technology. For example, if two players are equally likely to win a game of tennis on past performance, then a sequence of games between the two players could be simulated by successive tossing of a fair coin (heads player *A* wins, tails player *B* wins) or randomly selecting numbers from the list of natural numbers and noting whether the result is even (player *A* wins) or odd (player *B* wins).

skip counting

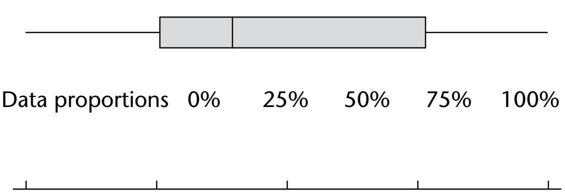
Counting from a given starting value using multiples of a fixed natural number; for example, {2, 4, 6, ...} or {7, 12, 17...}.

spread

This is a statistic that indicates how widely the values of a data set are distributed. Common measures of spread include *range*, *inter-quartile range*, *quantiles* and *percentiles* and *mean absolute difference*.

- **box-plot**: A form of data representation where the ends of a rectangular box are aligned on a numerical scale with a given proportion of a *sample* of uni-variate data. For example, the resting heartbeats of a group of athletes may be measured and a box-plot constructed to correspond to the middle 50% of values. Lines (whiskers) are added to show the lower and upper 25% of the data. The *median* value (the middle value of the sample) is also indicated by a vertical line parallel to the ends of the box:

  Box-plot of proportions of the sample



Scale of values being measured

- **inter-quartile range**: When a data set is ordered, inter-quartile range is the difference between the upper qaurter value and the lower quarter value (that is the difference between the 75th percentile and the 25th percentile).
- **outlier**: Informally an outlier is a value which lies outside the distribution of most of the values in a data set. For example, the height of a very tall or very short person would likely be an outlier in a data set of heights of randomly selected people. A characterisation of a likely outlier is a value that lies more than 1.5 times the inter-quartile range outside of the upper or lower quartiles.
- **stem-plot**: Also called a stem and leaf plot, a table where discrete data is represented (usually in order) by distinguishing values (the leaf) within set intervals (the stem). For example, the set of students heights in cms {152, 158, 159, 164, 164, 166, 169, 170, 172, 188} can be represented using a stem and leaf plot as:

Stem	Leaf
15	2 8 9
16	4 4 6 9
17	0 2
18	8

Key : 15|2 = 152 cms

Stem plots provide a visual indication of spread.

square number

An element of the set {1, 4, 9, 16, 25 ...}. A square number has an *odd* number of distinct elements in its factor set; for example, the factor set of 16 has five distinct elements: {1, 2, 4, 8, 16}.

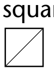
- **perfect square**: A number is a perfect square if it is the square of an integer or rational number; for example, 169 is a perfect square as $13^2 = 169$; similarly 0.81 is a perfect square as $(\frac{9}{10})^2 = 0.81$.

square root

The positive square root of a given real number x is the real number y such that $y^2 = x$. For example, the square root of 9 is 3. This is written symbolically as $\sqrt{9} = 3$. Originally, the square root was taken to refer to the side length (root) of a square whose area was a given positive number. Thus, a square of area 9 square units has a side length (square root) of 3 units. Most square roots are not rational numbers but irrational real numbers. For example, a square of area 2 has an exact side length of the square root of 2, or $\sqrt{2}$. This is approximately 1.4 units in length. Every positive real number has two square roots, one positive and one negative, for example, the square roots of 9 are 3 and –3. This is written symbolically as $\pm\sqrt{9} = -3$ and 3.

square root of 2

Is the irrational number, $\sqrt{2}$, whose value corresponds to the length of the diagonal of a unit square.

 1 unit

Its approximate value is 1.414 correct to 3 decimal places. The decimal expansion for the square root of two correct to 100 significant figures is:

1.41421356237309504880168872420969807856967187537694807317667973799073247846210703885038753432764157 3.

The digits in this decimal expansion do not display any recurring pattern, a property which distinguishes irrational numbers from rational numbers.

standard unit

A formal unit from a system of units which is comprehensive and is used to define other units or combinations of units. For example, in the metric system, the standard units for *length*, *mass* and *time* are respectively, *metre*, *kilogram* and *second*. The standard units are described in the International System of Units (SI). Related *formal*, units are:

centimetre = metre × $\frac{1}{100}$

tonne = kilogram × 1 000

minute = second × 60.

Other non-standard formal units are, for example, carat, gallon, hour and knot.

surd (quadratic)

An irrational number such as $\sqrt{2}$ or $(5+\sqrt{3})/4$. An older term of reference for a particular type of irrational number.

surface area

Area of the surfaces of a three-dimensional shape or object.

symbolic

Using marks or symbols that have a meaning particular to mathematical language, for example, the written statement ‘two is less than 3’ can be written symbolically as ‘2 < 3’.

symmetry

Property of regularity in shape by, for example, reflection or rotation. Thus the letter **T** is symmetrical by reflection, the letter **Z** is symmetrical by rotation, the letter **H** is symmetrical by both reflection and rotation, the letter **R** is *not* symmetrical.

- **asymmetry:** Irregular, does not display symmetry. The human body is asymmetrical with respect to an imaginary line ‘down the middle’.

tessellation

A repeated pattern in the plane or on a surface where shapes completely fill all of the space around a given point where their boundaries meet. For example, a honeycomb is a tessellation using hexagons Tiling patterns are tessellations using rectangular tiles or brick pavers in paths, mosaics in buildings, quilts and art.

transformation

A one-to-one correspondence of points in the plane. *Reflections*, *rotations* and *dilations* are examples of transformations.

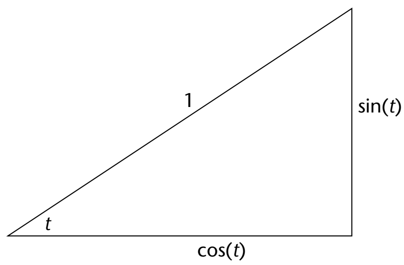
- **isometric**: Literally ‘the same measure’, an isometry is a transformation that leaves lengths, area and angles unchanged.

tree diagram

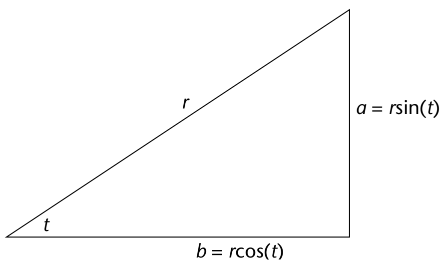
A diagram consisting of line segments connected like the branches and twigs of a tree used to indicate the relationship between sets or events, for example a family tree.

trigonomentry (sine, cosine and tangent)

In the following right angled triangle, with long side length 1 unit, the vertical side length is sin(*t*) and the horizontal side length is cos(*t*)



if the triangle is dilated by a factor *r* from the point at which the angle t is formed, then, by similarity:



and $\sin(t) = \frac{a}{r}$ $\cos(t) = \frac{b}{r}$ $\tan(t) = \frac{ar}{br} = \frac{a}{b}$

undefined term

A term or expression taken as accepted without definition. These are the basic building blocks of mathematics; for example, element and set are undefined terms in the *Structure* dimension, while point and line are undefined terms in the *Space* dimension. Undefined terms may be characterised by informal description, or illustrated by examples. Other mathematical terms and expressions are defined using undefined terms and relations on them.

union (set)

Given two sets *A* and *B*, their union, written $A \cup B$ is the set of all elements which occur in either set, listed without repetition. For example, if $A = \{ a, b, d, z \}$ and $B = \{ a, c, x, y, z \}$ then $A \cup B = \{ a, b, c, d, x, y, z \}$.

unit

A basic or fundamental construct for counting and/or measurement. For example, the number 1 is the unit for counting (from the Latin *unus* for one). The metre is the standard unit for measurement of length in the metric system.

uni-variate (data)

Data relating to measurement of a single variable, for example, shoe size.

variable

A term used to designate an *arbitrary* element of a set. For example, if n is any natural number, then $m = 2n + 1$ is an odd natural number. The terms *n* and *m* are called variables.

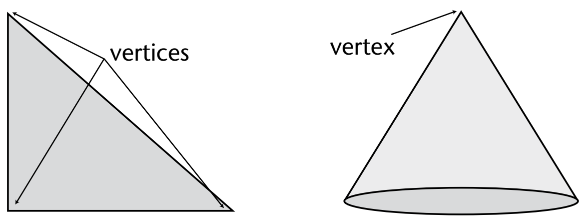
- **arbitrary (free) variable**: A variable whose scope is not limited by a logical quantifier. Free variables frequently are used in proofs to represent an arbitrary element of a set.

venn diagram

A diagram consisting of a small number of possibly overlapping circles used to indicate the relationship between elements of a set and given properties or attributes.

vertex (also node or corner)

A vertex is a point in the plane or in space where several edges meet, but do not extend beyond, for example the corners of a triangle or the point of a cone:



Web references

School of Mathematics and Statistics at the University of St Andrew's, Scotland
<http://www-history.mcs.st-and.ac.uk/history/>

This website provides topic, context, chronology and biographical historical references, and links to other history of mathematics sites.

Cut The Knot, Alex Bogomolny
<http://www.cut-the-knot.org/glossary/atop.shtml>

This website contains an extensive mathematical glossary, items of interest, mathematical games and puzzles and is a mathematics forum.

A Dictionary of Units, Frank Tapson
<http://www.ex.ac.uk/cimt/dictunit/dictunit.htm>

This website provides a comprehensive summary of many of the units of measurement in use around the world today, some units of historical interest and conversions into standard SI units. It also contains links to other sites related to units and measurement.

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