



Victorian Essential Learning Standards

CAS Technology Mathematics at Level 6

Part 2: CAS technology – a brief background



Published by

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

41 St Andrews Place East Melbourne Victoria 3002 Australia

TELEPHONE +61 3 9651 4300 FACSIMILE +61 3 9651 4324

EMAIL vcaa@edumail.vic.gov.au

WEB www.vcaa.vic.edu.au



2. CAS technology – a brief background

This section outlines the development of CAS technology in the broad historical context of the use of technology for computation.

CAS is an acronym for Computer Algebra System and was originally used to describe a computer program whose principal purpose was to carry out symbolic computation or *computer algebra*. The word ‘technology’ is a composite word formed from two classical Greek roots: *tekhnē* meaning an ‘art’ or a ‘skill’, from which we obtain the word ‘technique’, and *logia*, which refers to a process of ‘systematic application’. Technology can then be considered as the systematic application of an art or skill.

The equivalent Latin word is *ars* from which we derive the English ‘art’, in the sense of ‘the art of’. The old English term *arte* or *artes* is often used in historical texts to indicate a way of doing something; for example, Roberte Recorde’s sixteenth century books ‘The Grounde of Artes, teachings the Worke and Practise, of Arithmetike, both in whole numbers and fractions’ (1543) and ‘The Whetstone of Witte’, which is the second part of ‘Arithmetike’, containing the ‘Extraction of Rootes’, the ‘Cossike Practice’, with the ‘Rules of Equation’, and the ‘Woorkes of Surde Numbers’ (1557). The title of one of the main ancient Chinese mathematical texts from the first century BC is the ‘Nine Chapters on the Mathematical Art’. Here, the term ‘Art’ is used in much the same sense as that described above.

The Oxford English Dictionary describes ‘art’ as ‘a skill, especially a human skill in contrast to nature’ and as ‘certain branches of learning serving as intellectual instruments for more advanced studies’. A closely related word is ‘artefact’, which is described as ‘a product of human art and workmanship’. We often hear of archaeologists discovering various human *artefacts* at historical sites, and hypothesising about and investigating their use and significance. The archaeologist knows that these ‘things’ are ‘artefacts’ rather than mere ‘objects’ because of their practical or recreational association with human beings in a given situation and context. People often think of ‘technology’ in terms of particular artefacts, such as a mobile phone, computer or calculator; however, it is important to explicitly acknowledge the link between *tekhnē* or ‘art’ and a particular ‘artefact’. That is, *technology* is a synergy of *artefact* and *art* – the *systematic* application of the art of using a particular object as a tool for a purpose.

Throughout history, various technologies have been employed to support working mathematically, often in the area of numerical calculation. For example, tokens, pebbles, beads and other forms of ‘counter’ used with abacuses and counting board have been employed since antiquity, and continue to be used today. Indeed the terms ‘calculator’ and ‘calculus’ derive from the Latin word for a small stone (a *calculus*) used specifically for this purpose. From its popularisation in Europe by Leonardo Fibonacci in twelfth century Italy, the Hindu-Arabic number system, and related ‘pen and paper’ technology, for arithmetic computation was considered a radical and controversial innovation. It took several centuries to become generally accepted in merchant banking and accounting practices in renaissance Italy, and subsequently more broadly.

The late thirteenth and early fourteenth centuries was a golden age in Chinese mathematics. Tasks involving the formulation of problems using polynomial equations were set up and solved using the counting board. This method was based on the manipulation of coefficients of polynomial functions in rectangular arrays formed on the counting board, and was one of the earliest forms of *computational*

algebra. From the seventeenth century through to the 1930s, various mechanical devices for computation were developed. In 1937, these were replaced by electro-mechanical devices, and from the 1950s, by electronic devices.

In each context, the use of ‘technology’ has required the systematic application of an art in conjunction with a relevant *tool*, or *artefact*, which may be a diagram, schema, mechanical device or an electronic device. Table 1 summarises the calculating and computing devices that have been used throughout history.

Table 1: Summary of calculating and computing devices

Alternative	Desktop	Hand-held		
	Counting boards	Hand/finger calculators		
	Mechanical calculators and engines	Abacuses	Logarithms and slide-rules	
	Electro-mechanical computers			
	Electronic thermionic computers			
	Electronic digital semi-conductor computers		Electronic digital <i>arithmetic</i> calculators	
			Electronic digital <i>scientific</i> calculators	
			Electronic digital <i>graphics</i> calculators	
			Electronic digital computer algebra system (CAS) calculators	
Quantum computing, DNA computing and other computing platforms		‘palm-top’ convergent ICT devices, with various functionality, including computer algebra		

When technologies become familiar, their application is often taken for granted. It is usually when a *new* technology is developed, or a *new application* of an existing technology emerges, that we critically reflect on its current and future potential in various contexts.

Since the 1940s, terms such as ‘calculator’ and ‘computer’ have come to refer to the *artefact* of the *technology*, rather than the *person* who is *operating* the technology. Generally, technologies make our lives easier, enabling us to do certain things more readily and reliably with less effort. At times, new technologies replace old technologies, and sometimes they subsume them or exist alongside them.

Since the early 1970s, electronic digital technologies have developed specifically for mathematics and mathematics education. Early on there were *arithmetic* four-function calculators, then came *scientific*

calculators, various software applications on microcomputers, and *graphics* calculators. More recently hand-held calculators with CAS capabilities have emerged. The relationship between the corresponding *numerical*, *graphical* and *symbolic* functionalities of hand-held calculators is summarised in Figure 1.

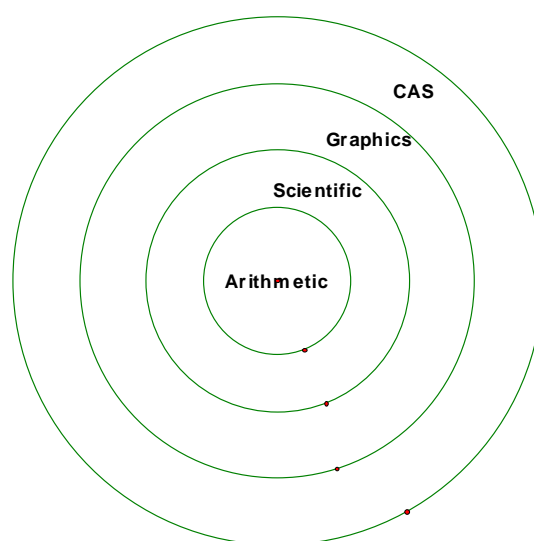


Figure 1: The relationship between functionalities of arithmetic, scientific, graphics and CAS calculators

While *arithmetic* and *scientific* calculators operate in the *numeric* mode, and *graphics* calculators operate in *numeric* and *graphic* modes, CAS calculators can operate in all three modes: *numeric*, *graphic* and *symbolic*, as shown in Figure 2.

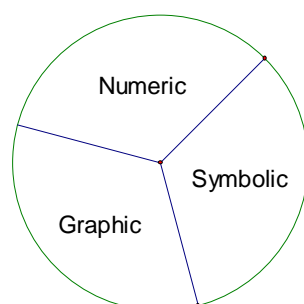


Figure 2: The three integrated modes for CAS

The first scientific calculators capable of *numerical* computation of arithmetic and scientific functions became available in the early 1970s. From 1978, these were permitted in the end of secondary schooling examinations in Victoria. Calculators with *numerical* and *symbolic* computation became available in 1986, but have never been permitted in the end of secondary schooling examinations in Victoria. Calculators with *numerical* and *graphical* computation; that is *graphics calculators*, became available in 1989, and were first permitted in the end of secondary schooling examinations in Victoria in 1997. Calculators with *numerical*, *graphical* and *symbolic* computation; that is CAS calculators, have been available since 1992, and, following the Victorian Curriculum and Assessment Authority pilot study, were first permitted in relevant end of secondary schooling mathematics examinations in Victoria in 2006.

The first high-level programming language for electronic digital computers, FORTRAN (FORMula TRANslation), was developed by a team of IBM programmers in 1954 and released in 1957. Subsequently, another high-level language, ALGOL, released in 1958, was developed by an international committee to describe computational processes. It was initially called International Algebraic Language (IAL), but this was later changed to ALGOL (ALGORithmic Language). ALGOL was updated in 1960 and 1968. The programming language C, released in 1975, and its variants, are derivatives of ALGOL. From the start, various programming languages were developed to have symbolic (algebraic) manipulation facilities. Early CAS were programs developed specifically for the purpose of *symbolic computation* implemented by computers, and typically utilised in specialised research contexts. Today, some *specialised* CAS still serve this function.

Platforms for sophisticated and integrated collections of algorithms began to be seriously developed in the latter part of the 1970s. These platforms can operate freely on mathematical constructs and enable the user to define and manipulate mathematical objects much as they would do mentally or by hand. Possibly the most well known and widely available of these are the various *general* CAS such as *Derive*, *Maple* and *Mathematica*. Contemporary versions of these are implemented as programs on computers, with integrated numerical, statistical, graphical and symbolic functionalities and the capacity to incorporate graphs, computations and texts in a single document. Other software with substantive and purposefully designed mathematical capabilities includes spreadsheets, statistical analysis systems, and dynamic geometry systems such as *Cabri-Geomètre* or *Geometer's Sketchpad*. These are constructed using special purpose programming languages, but their interfaces are designed to be readily utilised by non-experts. Modern CAS utilise a sophisticated and integrated collection of programs and incorporate their own programming language.

During the 1990s, several processes led to the development of 'hand-held' CAS calculators. These included:

- the use of adapted versions of the CAS *Derive* to provide symbolic manipulation functionality for the Texas instruments TI-92 family and TI-89 families of hand-held calculators
- the development and implementation of propriety symbolic manipulation functionality for particular models of Hewlett-Packard (HP) and CASIO hand-held calculators
- the increasing sophistication of 'hand-held' calculators and convergence between 'hand-held' and 'desktop' platform and functionality
- increased memory capacity of hand-held devices.

Since the late 1990s, hand-held devices (graphics and CAS calculators) have had substantially increased memory capacity. This has led to the incorporation of various spreadsheet, geometric (for example, *Cabri-Geomètre* or *Geometer's SketchPad*.) and text functionalities as well as the rapid growth in development, and student use, of programs and applications (including quasi-CAS) in calculator programming languages. These programs and applications carry out algebraic manipulations (for example, *Factor 9*), the solution of equations and symbolic calculus routines such as differentiation (for example, *Symbolic*), and some classes of anti-differentiation problems. They can also be used for more involved numerical and/or graphical computations such as the numerical solution of differential equations using Euler's methods and to draw direction (slope) fields for differential equations.

The most recent types of hand-held CAS calculators, such as the *Classpad* 300 series and TI-*nspire* CAS+, have been developed to combine various numerical, statistical, graphical, symbolic, geometric and text functionalities within a dynamic operating system. TI-*nspire* CAS (the software version does not have the CAS+ designation) operates explicitly on both hand-held and computer-based (software) platforms. The *Classpad* 300 series and the HP series of hand-held CAS calculators work with these platforms via an emulator.

Thus *symbolic manipulation* (or computer algebra in the original sense) is only *one* of these calculators' capabilities, even though it has been designed as an *integral part* of the technology application. These devices have been designed and developed as technologies that are more than just 'calculators' that can 'do algebra'. They can also be used for numerical, graphical, statistical, geometric and symbolic computations, constructions and applications in and across the *Number, Space, Measurement, chance and data, Structure* and *Working mathematically* dimensions of the Mathematics domain of the Victorian Essential Learning Standards (VELS) and in senior secondary mathematics studies.