Victorian Essential Learning Standards

CAS Technology Mathematics at Level 6

Part 3: CAS-related activities for dimensions

Contents

Victoria The Place To Be

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VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

41 St Andrews Place East Melbourne Victoria 3002 Australia

TELEPHONE +61 3 9651 4300 FACSIMILE +61 3 9651 4324

EMAIL vcaa@edumail.vic.gov.au

WEB www.vcaa.vic.edu.au

3. CAS-related activities for dimensions

CAS and Number

Following are some CAS-related activities for exploring number concepts, skills and processes related to elements of the Level 6 standards for the *Number* dimension of the Mathematics VELS.

Number

At Level 6, students comprehend the set of real numbers containing natural, integer, rational and irrational numbers. They represent rational numbers in both fractional and decimal (terminating and infinite recurring) forms (for example, $1\frac{4}{25} = 1.16$, $0.\overline{47} = \frac{47}{99}$). They comprehend that irrational numbers have an infinite non-terminating decimal form. They specify decimal rational approximations for square roots of primes, rational numbers that are not perfect squares, the golden ratio φ, and simple fractions of π correct to a required decimal place accuracy.

Students use the Euclidean division algorithm to find the greatest common divisor (highest common factor) of two natural numbers (for example, the greatest common divisor of 1071 and 1029 is 21 since $1071 = 1029 \times 1 + 42$, $1029 = 42 \times 24 + 21$ and $42 = 21 \times 2 + 0$).

Students carry out arithmetic computations involving natural numbers, integers and finite decimals using mental and/or written algorithms (one- or two-digit divisors in the case of division). They perform computations involving very large or very small numbers in scientific notation (for example, $0.0045 \times 0.000028 = 4.5 \times 10^{-3} \times 2.8 \times 10^{-5} = 1.26 \times 10^{-7}$).

They carry out exact arithmetic computations involving fractions and irrational numbers such as square roots (for example, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$ and multiples and fractions of π (for example, $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$). They use appropriate estimates to evaluate the reasonableness of the results of calculations involving rational and irrational numbers, and the decimal approximations for them. They carry out computations to a required accuracy in terms of decimal places and/or significant figures.

Element of Number dimension

They represent rational numbers in both fractional and decimal (terminating and infinite recurring) forms (for example, $1\frac{4}{25} = 1.16$, $0.\overline{47} = \frac{47}{99}$).

Activities include:

1. Generate decimal equivalents for $\frac{1}{n}$ for n from 2 to 9 and then generate corresponding values of $\frac{m}{n}$ for m from 2 to $n - 1$, predicting values after the first few using the relationship $\frac{m}{n} = m \times \frac{1}{n}$; for example, where $n = 7$. Extension to cases where $m > n$, and for other values of n; for example, $\frac{15}{11} = 15 \times 0.090909... = 1.363636...$

2. Express a range of given fractions as a sum of distinct unit fractions (that is, fractions with numerator one, and no repeats permitted) and check the result. This form is called an Egyptian fraction since it was the form used for calculation and problem solving in ancient Egypt. A process for the required decomposition is to *subtract* the largest available unit fraction that leaves a positive difference and repeat this process until a unit fraction result is obtained. The given fraction can then be reconstructed as the sum of the unit fractions used to subtract and the final resultant unit fraction. For example, $\frac{13}{16} = \frac{1}{2} + \frac{1}{4} + \frac{1}{16}$ since $\frac{13}{16} - \frac{1}{2} = \frac{5}{16}$ and $\frac{5}{16} - \frac{1}{4} = \frac{1}{16}$. Students should be allowed to develop a process of their own rather than be given an algorithm and explore whether there is a unique expression of this kind for any given fractions and, if not, how to determine a shortest form.

3. Explore the pattern $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$, $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$, ... to find the general result for $\frac{1}{n} + \frac{1}{n+1}$ (the resultant *numerator* is the *sum* of the consecutive values n and n+1 while the resultant *denominator* is their *product*). This can be extended to $\frac{1}{n} + \frac{1}{m}$ where n and m are distinct non-zero whole numbers and then to the general form for addition of fractions $\frac{p}{n} + \frac{q}{m}$.

4. Investigate continued fraction representation of rational numbers, for example the continued fraction form of $\frac{17}{12} = 1\frac{5}{12}$ is $1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}$.

5. Obtain rational numbers in fraction form from the corresponding infinite recurring decimal expansions (infinite geometric series). This can be done by consideration of the convergence of the partial sums of the corresponding geometric sequence numerically, graphically and algebraically (for example, $0.\overline{32} = 0.323232\ldots = \frac{32}{100} + \frac{32}{10000} + \frac{32}{1000000} + \ldots$) and by assigning an unknown constant, such as k, to the infinite series, multiplying by the corresponding power of ten, then subtracting and dividing to obtain the corresponding fractional form (for example, if $k = 0.323232\ldots$ then $100k = 32.323232\ldots$ and $99k = 32$ so $k = \frac{32}{99}$; generally if the sequence of digits before repetition is of length n, then k is multiplied by 10^n before subtraction and division).

Element of Number dimension

They comprehend that irrational numbers have an infinite non-terminating decimal form. They specify decimal rational approximations for square roots of primes, rational numbers that are not perfect squares, the golden ratio φ, and simple fractions of π correct to a required decimal place accuracy.

Activities include:

1. List rational approximations to numbers such as $\sqrt{2}$, π and φ , and note that their decimal representations to a given finite number of decimal places do not appear to contain any recurring sub-sequences (no *counter-examples* can be found in a finite set of test cases).

2. Develop proofs for the irrationality of certain real numbers such as $\sqrt{2}$, φ and x where $2^x = 5$. These proofs are all proofs by contradiction. For $\sqrt{2}$, the proof shows that the assumption of rationality leads to the contradiction of a fraction already expressed in simplest terms has having numerator and denominator with common factor, 2. For φ , geometric considerations on sides of a nested sequence of rectangles are used to show that it is not possible for the required sides of a rectangle whose ratio of lengths is φ to be co-measurable by positive integers – one runs out of possible pairs of *positive* integers after a finite number of steps. For x where $2^x = 5$ and hence $x = \log_2(5)$, a contradiction is obtained by showing that the assumption of rationality leads to an odd number being equal to an even number – if x is rational, that is of the form $x = \frac{m}{n}$ where m and n are positive integers and $m > n$, then $2^x = 5$ implies $2^{\frac{m}{n}} = 5$ hence $2^m = 5^n$, but 2^m will be *even* and 5^n will be *odd* – the assumption of rationality is not true.

Element of Number dimension

Students use the Euclidean division algorithm to find the greatest common divisor (highest common factor) of two natural numbers (for example, the greatest common divisor of 1071 and 1029 is 21 since $1071 = 1029 \times 1 + 42$, $1029 = 42 \times 24 + 21$ and $42 = 21 \times 2 + 0$).

Activities include:

1. Find greatest common divisors, *gcd*, (highest common factors) for a set of natural numbers (for example, 43258 and 596) by using lists of factors and prime factors of these numbers (such as factor trees) and using technology *gcd* functionality. If students are asked to find a systematic method of their own *without* access to such a list, they are likely to do some background work on the development of the *Euclidean division algorithm*.

2. Development and application of the *Euclidean division algorithm* for finding the *gcd* of two natural numbers (there are both repeated-subtraction and division forms of the algorithm. The algorithm is also closely related to *continued fractions*.

Element of Number dimension

They carry out exact arithmetic computations involving fractions and irrational numbers such as square roots (for example, $\sqrt{18} = 3\sqrt{2}$, $\sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$) and multiples and fractions of π (for example $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$).

Activities include:

1. Simplify quadratic surds (for example, $\sqrt{96} = \sqrt{16 \times 6} = \sqrt{16} \times \sqrt{6} = 4\sqrt{6}$) and explore the arithmetic of surd terms obtained from geometric considerations (Pythagoras' theorem) or those obtained from the solution of quadratic equations (verification of solutions, for example, $2x^2 - 5x + 1 = 0$) and factorisation of quadratics over the real field (for example, $x^2 + 4x - 11$).

2. Manipulate sums and differences of multiples and fractions of π (for example, $n\pi \pm \frac{\pi}{3}$ where n is an integer; generalisation from specific cases to consideration of $\frac{\pi}{a} + \frac{\pi}{b} = \frac{(a+b)\pi}{ab}$ where a and b are non-zero integers).

CAS and Space

Following are some CAS-related activities for exploring number concepts, skills and processes related to elements of the Level 6 standards for the *Space* dimension of the Mathematics VELS.

Space

At Level 6, students represent two- and three-dimensional shapes using lines, curves, polygons and circles. They make representations using perspective, isometric drawings, nets and computer-generated images. They recognise and describe boundaries, surfaces and interiors of common plane and three-dimensional shapes, including cylinders, spheres, cones, prisms and polyhedra. They recognise the features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use associated angle properties.

Students explore the properties of spheres.

Students use the conditions for shapes to be congruent or similar. They apply isometric and similarity transformations of geometric shapes in the plane. They identify points that are invariant under a given transformation (for example, the point (2, 0) is invariant under reflection in the x-axis, so the x axis intercept of the graph of $y = 2x - 4$ is also invariant under this transformation). They determine the effect of changing the scale of one characteristic of two- and three-dimensional shapes (for example, side length, area, volume and angle measure) on related characteristics.

They use latitude and longitude to locate places on the Earth's surface and measure distances between places using great circles.

Students describe and use the connections between objects/location/events according to defined relationships (networks).

CAS calculators typically have functionality that enables them to be used to draw and manipulate representations of various two- and three-dimensional shapes. They can be used to provide these representations by a combination of plotting points, lines, line segments, curves, regions and surfaces using two- and three-dimensional graphics functionality based on various coordinate systems (rectangular, polar, spherical and parametric). For three-dimensional shapes, surface components may be 'opaque' (shaded or coloured) or 'transparent' (only the edges of surface polygons are shown in 'wire-frame' view). The point of view and viewing perspective of three-dimensional shapes can also be varied, sometimes dynamically; projections obtained from a three-dimensional perspective onto various planes (from x-y-z to x-y, y-z and x-z planes respectively) or a contour map formed. Some CAS calculators also incorporate dynamic geometry software (such as versions of *Cabri-Geometry*, *Geometer's Sketchpad* or similar) as part of their functionality.

Element of Space dimension

Students represent two- and three-dimensional shapes using lines, curves, polygons and circles. They make representations using perspective, isometric drawings, nets and computer-generated images. They recognise and describe boundaries, surfaces and interiors of common plane and three-dimensional shapes, including cylinders, spheres, cones, prisms and polyhedra.

Activities include:

1. Construct various two-dimensional shapes using points and line segments (this can be done *with* and *without* coordinates systems if dynamic geometry functionality is also available) and identify necessary and sufficient conditions for construction of given shapes with specified dimensions.

2. Construct various curves, surfaces, and shapes in three dimensions and explore their projections with respect to *x-y*, *y-z* and *x-z* planes.

Element of Space dimension

They recognise the features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and use associated angle properties.

Activities include:

1. Apply dynamic geometry functionality to carry out constructions related to circles, segments and tangents and carry out related measurements, using built-in measure functions to exemplify angle properties.

2. Develop geometric proofs of angle properties.

Element of Space dimension

Students use the conditions for shapes to be congruent or similar. They apply isometric and similarity transformations of geometric shapes in the plane.

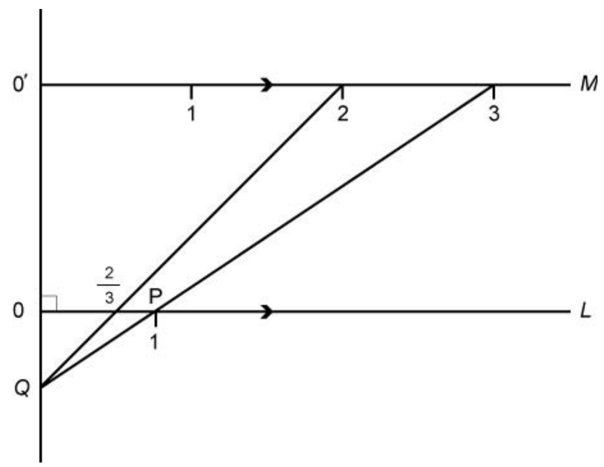
Activities include:

1. Apply dynamic geometry functionality to construct shapes according to specification; for example, different methods for construction of a kite or construction of a regular pentagon.

2. Develop geometric proofs for constructions.

2. Apply dynamic geometry functionality and use scale and similarity to locate a set of fractions on a *common* number line (Draw a horizontal line L, with a fixed origin O, which corresponds to the number ‘0’ at its left-hand end, and choose a point P to the right of the origin so that the segment formed designates a unit length. That is, the right-hand endpoint of this segment corresponds to the number ‘1’. Construct a perpendicular to L, through O and mark a point O' on this line, vertically above O. Use this point to construct a new line M, parallel to L. Using the point O' vertically above O as the origin on M, construct a unit segment for M, different from, and longer than, the unit chosen for L. To locate the point corresponding to the fraction $\frac{a}{b}$, where $0 < a < b$, on L, mark off b copies of the unit for M to the right of O', and draw a line from the endpoint of these copies through P until it meets the perpendicular (this will be below L) at a point Q. Then mark off a copies of the unit for M to the right of O', and draw a line from the endpoint of these copies through the line L to the point Q. Where this line intersects L will be the required point corresponding to the endpoint of the line segment (on L) that is $\frac{a}{b}$ of the unit on L, from O).

Consider $\frac{2}{3}$ for example,



Element of Space dimension

They identify points that are invariant under a given transformation (for example, the point (2, 0) is invariant under reflection in the x-axis, so the x axis intercept of the graph of $y = 2x - 4$ is also invariant under this transformation).

Activities include:

1. Use graphing functionality and/or dynamic geometry functionality to explore which points on a curve or shape are invariant (that is, are their own image) under a given transformation of the plane. For example, the x-axis intercepts of the graph of a quadratic function are invariant under dilation from the horizontal axis; however, its vertex, in general, is not (unless there happens to be exactly one intercept, in which case it *is* the vertex).

Element of Space dimension

They determine the effect of changing the scale of one characteristic of two- and three-dimensional shapes (for example, side length, area, volume and angle measure) on related characteristics.

1. Apply dynamic geometry functionality to construct shapes according to specification. Vary the scale of one of the dimensions of a two- or three-dimensional object and use built-in measure functionality to explore the corresponding effect on perimeter, area, surface area or volume.

2. Use formulas and table functionality, as applicable, to vary the scale of one of the dimensions of a two- or three-dimensional object, and calculate the corresponding effect on perimeter, area, surface area or volume.

CAS and Measurement, chance and data

Following are some CAS-related activities for exploring number concepts, skills and processes related to elements of the Level 6 standards for the *Measurement, chance and data* dimension of the Mathematics VELS.

Measurement, chance and data

At Level 6, students estimate and measure length, area, surface area, mass, volume, capacity and angle. They select and use appropriate units, converting between units as required. They calculate constant rates such as the density of substances (that is, mass in relation to volume), concentration of fluids, average speed and pollution levels in the atmosphere. Students decide on acceptable or tolerable levels of error in a given situation. They interpret and use mensuration formulas for calculating the perimeter, surface area and volume of familiar two- and three-dimensional shapes and simple composites of these shapes. Students use pythagoras theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles.

They use degrees and radians as units of measurement for angles and convert between units of measurement as appropriate.

Students estimate probabilities based on data (experiments, surveys, samples, simulations) and assign and justify subjective probabilities in familiar situations. They list event spaces (for combinations of up to three events) by lists, grids, tree diagrams, venn diagrams and karnaugh maps (two-way tables). They calculate probabilities for complementary, mutually exclusive, and compound events (defined using *and*, *or* and *not*). They classify events as dependent or independent.

Students comprehend the difference between a population and a sample. They generate data using surveys, experiments and sampling procedures. They calculate summary statistics for centrality (mode, median and mean), spread (box plot, inter-quartile range, outliers) and association (by-eye estimation of the line of best fit from a scatter plot). They distinguish informally between association and causal relationship in bi-variate data, and make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables.

Element of Measurement, chance and data dimension

They calculate constant rates such as the density of substances (that is, mass in relation to volume), concentration of fluids, average speed and pollution levels in the atmosphere. Students decide on acceptable or tolerable levels of error in a given situation. They interpret and use mensuration formulas for calculating the perimeter, surface area and volume of familiar two- and three-dimensional shapes and simple composites of these shapes.

Activities include:

Calculate various rates from formulas and construct corresponding rate graphs (for example, $density = \frac{mass}{volume}$; $average\ speed = \frac{distance\ travelled}{time\ taken}$; $concentration = \frac{grams\ of\ chemical}{litres\ of\ water}$), to find amounts required to make up given quantities of a substance from its constituents.

2. Calculate various measures from formulas and given data, (for example, the surface area and volume of a capsule formed by a cylinder and two hemi-spheres in terms of radius and length) and draw corresponding variation graphs.

Element of Measurement, chance and data dimension

Students use pythagoras theorem and trigonometric ratios (sine, cosine and tangent) to obtain lengths of sides, angles and the area of right-angled triangles.

They use degrees and radians as units of measurement for angles and convert between units of measurement as appropriate.

Activities include:

1. Complete computations on the lengths and areas of right-angle triangles, given sufficient information and calculation of lengths and related surface area and volumes of three-dimensional shapes.

2. Investigate Pythagorean triples; that is, ordered triples of whole non-zero numbers (a, b, c) that satisfy the relation $a^2 + b^2 = c^2$ (this activity should follow on from consideration of proofs of Pythagoras' theorem).

3. Use scaling from right-angle triangles of unit hypotenuse length (freestanding or within a unit circle), or ratios of side lengths in right-angle triangles, to evaluate sine, cosine and tangent, and draw the corresponding graphs using degree and radian measures for the horizontal scale. Complete related computations involving angles and lengths, including application of Pythagoras' theorem to sine and cosine in the case of a right-angle triangle with unit hypotenuse. Convert between degrees and radians and apply the corresponding relation to computation of lengths of arcs of circles of a given radius and draw corresponding conversion graphs.

4. Use dynamic geometry functionality to explore the geometric relationship between the area of the squares formed on the sides of triangles and the measure of the angle subtended, where $c^2 < a^2 + b^2$ (acute angle), $c^2 = a^2 + b^2$ (right angle), $c^2 > a^2 + b^2$ (obtuse angle).

Element of Measurement, chance and data dimension

They calculate summary statistics for centrality (mode, median and mean), spread (box plot, inter-quartile range, outliers) and association (by-eye estimation of the line of best fit from a scatter plot). They distinguish informally between association and causal relationship in bi-variate data, and make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables.

Activities include:

1. Calculate summary statistics for a range of uni-variate and bi-variate data and represent these using tables, diagrams and graphs. For a systematic approach to finding a line of best fit, use a simple method with two means.

CAS and Structure

Following are some CAS-related activities for exploring number concepts, skills and processes related to elements of the Level 6 standards for the *Structure* dimension of the Mathematics VELS.

Structure

At Level 6, students classify and describe the properties of the real number system and the subsets of rational and irrational numbers. They identify subsets of these as discrete or continuous, finite or infinite and provide examples of their elements and apply these to functions and relations and the solution of related equations.

Student express relations between sets using membership, $\in$, complement, $'$, intersection, $\cap$, union, $\cup$, and subset, $\subseteq$, for up to three sets. They represent a universal set as the disjoint union of intersections of up to three sets and their complements, and illustrate this using a tree diagram, venn diagram or karnaugh map.

Students form and test mathematical conjectures; for example, ‘What relationship holds between the lengths of the three sides of a triangle?’

They use irrational numbers such as, π, φ and common surds in calculations in both exact and approximate form.

Students apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computation with number, to rearrange formulas, rearrange and simplify algebraic expressions involving real variables. They verify the equivalence or otherwise of algebraic expressions (linear, square, cube, exponent, and reciprocal, (for example,

$4x - 8 = 2(2x - 4) = 4(x - 2)$; $(2a - 3)^2 = 4a^2 - 12a + 9$; $(3w)^3 = 27w^3$; $\frac{x^3y}{xy^2} = x^2y^{-1}$; $\frac{4}{xy} = \frac{2}{x} \times \frac{2}{y}$).

Students identify and represent linear, quadratic and exponential functions by table, rule and graph (all four quadrants of the Cartesian coordinate system) with consideration of independent and dependent variables, domain and range. They distinguish between these types of functions by testing for constant first difference, constant second difference or constant ratio between consecutive terms (for example, to distinguish between the functions described by the sets of ordered pairs {(1, 2), (2, 4), (3, 6), (4, 8) …}; {(1, 2), (2, 4), (3, 8), (4, 14) …}; and {(1, 2), (2, 4), (3, 8), (4, 16) …}). They use and interpret the functions in modelling a range of contexts.

They recognise and explain the roles of the relevant constants in the relationships $f(x) = ax + c$, with reference to gradient and y axis intercept, $f(x) = a(x + b)^2 + c$ and $f(x) = ca^x$.

They solve equations of the form $f(x) = k$, where k is a real constant (for example, $x(x + 5) = 100$) and simultaneous linear equations in two variables (for example, $\{2x - 3y = -4$ and $5x + 6y = 27\}$ using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods.

Element of Structure dimension

Students apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computation with number, to rearrange formulas, rearrange and simplify algebraic expressions involving real variables. They verify the equivalence or otherwise of algebraic expressions (linear, square, cube, exponent, and reciprocal, (for example,

$4x - 8 = 2(2x - 4) = 4(x - 2)$; $(2a - 3)^2 = 4a^2 - 12a + 9$; $(3w)^3 = 27w^3$; $\frac{x^3 y}{xy^2} = x^2y^{-1}$; $\frac{4}{xy} = \frac{2}{x} \times \frac{2}{y}$).

Activities include:

1. Use symbolic manipulation functionality (*expand, factorise, collect, simplify, solve, eliminate*) to verify equivalences (for example, relationships between coefficients of quadratic trinomials expressions and factorised forms over the set of integers, Z, rationals, Q, or reals, R, and related roots of quadratic equations); explore the notion of simplification (for example, any sum or difference of linear terms can be reduced to a single expression of the form $ax + b$), products, powers and quotients involving terms expressed in $base^{power}$ form; and re-arrange expressions (for example, in the formula for body mass index: $BMI = \frac{weight}{height^2}$).

Element of Structure dimension

Students identify and represent linear, quadratic and exponential functions by table, rule and graph (all four quadrants of the cartesian coordinate system) with consideration of independent and dependent variables, domain and range. They distinguish between these types of functions by testing for constant first difference, constant second difference or constant ratio between consecutive terms (for example, to distinguish between the functions described by the sets of ordered pairs {(1, 2), (2, 4), (3, 6), (4, 8) …}; {(1, 2), (2, 4), (3, 8), (4, 14) …}; and {(1, 2), (2, 4), (3, 8), (4, 16) …}). They use and interpret the functions in modelling a range of contexts.

Activities include:

1. Define the rule of various functions, construct corresponding tables of values and draw graphs (for domain and range, and the difference between these and plot-window specifications).

2. Generate lists and plot graphs of patterns and sequences using recursion relationships (for example, linear (arithmetic) and exponential (geometric), harmonic, lucas and fibonnacci sequences.

3. Use lists to systematically test consecutive values of data (list, ordered pairs or table) from various contexts for linear, quadratic or exponential relationships.

Element of Structure dimension

They recognise and explain the roles of the relevant constants in the relationships $f(x) = ax + c$, with reference to gradient and y axis intercept, $f(x) = a(x + b)^2 + c$ and $f(x) = ca^x$.

Activities include:

1. Generate families of graphs based on variation in one parameter of the rule of a given function (this can also be done using dynamic geometry functionality). Identify the effect on the general shape and location of the relevant function and the specification of key points and asymptote as applicable.

Element of Structure dimension

They solve equations of the form $f(x) = k$, where k is a real constant (for example, $x(x + 5) = 100$) and simultaneous linear equations in two variables (for example, $\{2x - 3y = -4$ and $5x + 6y = 27\}$ using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods.

Activities include:

1. Use the *solve* functionality to determine the (analytic) solution to various equations formed from rules for linear, quadratic, hyperbolic and exponential functions, and mensuration formulas.

2. Use tables for systematic guess, check and refine and bisection approaches to obtaining numerical solutions to various equations.

3. Use continued fractions (a recursively generated sequence) to approximate the solution to the roots of a quadratic equation (for example, $x^2 - 2x - 1 = 0$ has $1 + \sqrt{2}$ as one of its roots, which can be approximated by re-writing the original equation as $x^2 = 2x + 1 \Rightarrow x = 2 + \frac{1}{x}$ and using the corresponding recurrence relationship $x_{n+1} = 2 + \frac{1}{x_n}$ with a suitable choice for x_0, for example a value of 1.5.

4. Solve a 2 by 2 system of simultaneous linear equations using matrices and explore the general behaviour of such systems.

5. Use the *solve* functionality to determine the (analytic) solution to various simultaneous equations for linear-linear, linear-quadratic and linear-hyperbola combinations.

CAS and Working mathematically

Following are some CAS-related activities for exploring number concepts, skills and processes related to elements of the Level 6 standards for the *Working mathematically* dimension of the Mathematics VELS.

Working mathematically

At Level 6, students formulate and test conjectures, generalisations and arguments in natural language and symbolic form (for example, 'if m^2 is even then m is even, and if m^2 is odd then m is odd'). They follow formal mathematical arguments for the truth of propositions (for example, 'the sum of three consecutive natural numbers is divisible by 3').

Students choose, use and develop mathematical models and procedures to investigate and solve problems set in a wide range of practical, theoretical and historical contexts (for example, exact and approximate measurement formulas for the volumes of various three dimensional objects such as truncated pyramids). They generalise from one situation to another, and investigate it further by changing the initial constraints or other boundary conditions. They judge the reasonableness of their results based on the context under consideration.

They select and use technology in various combinations to assist in mathematical inquiry, to manipulate and represent data, to analyse functions and carry out symbolic manipulation. They use geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

Element of Working mathematically dimension

At Level 6, students formulate and test conjectures, generalisations and arguments in natural language and symbolic form (for example, 'if m^2 is even then m is even, and if m^2 is odd then m is odd'). They follow formal mathematical arguments for the truth of propositions (for example, 'the sum of three consecutive natural numbers is divisible by 3').

Activities include:

1. Use tables of values and other functionality to test conjectures values (for example, the conjecture that $n! + 1$ is always prime, or that the sum of n consecutive numbers is divisible by n).

2. Extend the search for primes to the proof (Euclid's proof) that, even though $n! + 1$ is not itself necessarily prime, there will be a new prime between n and $n! + 1$.

Element of Working mathematically dimension

They use geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

Activities include:

Use matrices to transform sets of points in the plane (for example, a unit square or circle, a straight line, the basic parabola) and graph original and image points on the same set of axes. Find those points which are self-images (that is, invariant) under a given transformation.